2024 Compt.	65/5/2	2
2024 Annual	65	B
2024 Annual	65/1/1	1
2024 Annual	65/1/2	2
2024 Annual	65/1/3	3
2024 Annual	65/2/1	1
2024 Annual	65/2/2	2
2024 Annual	65/2/3	3
2024 Annual	65/3/1	1
2024 Annual	65/3/2	2
2024 Annual	65/3/3	3
2024 Annual	65/4/1	1
2024 Annual	65/4/2	2
2024 Annual	65/4/3	3
2024 Annual	65/5/1	1
2024 Annual	65/5/2	2
2024 Annual	65/5/3	3
2023 Compt.	65/C/1	1
2023 Compt.	65/C/2	2
2023 Compt.	65/C/3	3
2023 Compt.	65	B
2023 Annual	65	B
2023 Annual	65/1/1	1
2023 Annual	65/1/2	2
2023 Annual	65/1/3	3
2023 Annual	65/2/1	1
2023 Annual	65/2/2	2
2023 Annual	65/2/3	3

2023 Annual	65/3/1	1
2023 Annual	65/3/2	2
2023 Annual	65/3/3	3
2023 Annual	65/4/1	1
2023 Annual	65/4/2	2
2023 Annual	65/4/3	3
2023 Annual	65/5/1	1
2023 Annual	65/5/2	2
2023 Annual	65/5/3	3
2022 Compt.	65/6/1	1
2022 Compt.	65/6/2	2
2022 Compt.	65/6/3	3
2022 Compt.	65/B/6	6
2022 Annual	65/1/1	1
2022 Annual	65/1/2	2
2022 Annual	65/1/3	3
2022 Annual	65/2/1	1
2022 Annual	65/2/2	2
2022 Annual	65/2/3	3
2022 Annual	65/3/1	1
2022 Annual	65/3/2	2
2022 Annual	65/3/3	3
2022 Annual	65/4/1	1
2022 Annual	65/4/2	2
2022 Annual	65/4/3	3
2022 Annual	65/5/1	1
2022 Annual	65/5/2	2
2022 Annual	65/5/3	3
2022 Annual	65/B/5	5

2024 Compt.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

*This section comprises 20 Multiple Choice Questions (MCQs) carrying **1** mark each.* $20\times1=20$

1. If A and B are two square matrices of order 2 and $|A| = 2$ and $|B| = 5$, then $|-3AB|$ is :

(A) -90 (B) -30

(C) 30 (D) 90

2. $\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to :

(A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$

(C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$

3. The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$

(C) 2 (D) 4

4. What is the value of $\dfrac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$

for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$

(C) $\frac{4}{3}$ (D) $\frac{4}{7}$

5. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\cdot\vec{b} > 0$ and $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

6. For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is :

(A) 4B (B) B

(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$

7. If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx}$ is equal to :

(A) 2at (B) $\frac{1}{t}$

(C) $-\frac{1}{t^2}$ (D) $-\frac{1}{2at^3}$

8. The function $f(x) = |x| - x$ where $x \in R$ is :

(A) continuous and differentiable at $x = 0$

(B) continuous but not differentiable at $x = 0$

(C) not continuous but differentiable at $x = 0$

(D) neither continuous nor differentiable at $x = 0$

9. If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is :

(A) 6
(B) 10
(C) 15
(D) 35

10. The number of discontinuities of the function f given by

$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$$

is :

(A) 0
(B) 1
(C) 2
(D) 3

11. $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is equal to :

(A) $\frac{1}{4} \log (x^4 + 1) + C$

(B) $\frac{1}{4} \log \left(\frac{1}{x^4 + 1} \right) + C$

(C) $\frac{x^3}{x^4 + 1} + C$

(D) $\frac{e^x}{x^4 + 1} + C$

12. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

(A) $-\frac{1}{64}$
(B) $-\frac{1}{32}$
(C) -32
(D) -64

13. If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :

(A) $\frac{1}{\pi}$ (B) π

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

14. A particular solution of the differential equation $x\frac{dy}{dx} + y = 0$, when $x = 1$ and $y = 1$, is :

(A) $y = x$

(B) $y = e^x$

(C) $y = \frac{1}{x}$

(D) $y = \log x$

15. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

16. A vector makes equal angles with positive directions of x, y and z axes. The direction cosines of the vector are :

(A) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(B) $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(C) $\frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}$

(D) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

17. The vector equation of the line passing through the points (0, 0, 2) and (3, – 2, 5) is :

(A) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 5\hat{k})$

(B) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$

(C) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 3\hat{k})$

(D) $\vec{r} = 3\hat{i} - 2\hat{j} + 5\hat{k} + \lambda(2\hat{k})$

18. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :

(A) $\frac{2\pi}{3}$ cm/s (B) π cm/s

(C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

20. *Assertion (A) :* $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

SECTION B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate :

$$\sin^{-1}\left(\sin\frac{13\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{3}\right) + \tan^{-1}(\sqrt{3})$$

22. Given that $f(x) = \dfrac{\log x}{x}$, find the point of local maximum of f(x).

23. (a) Find :

$$\int \frac{x^3 - 1}{x^3 - x}\, dx$$

OR

(b) Evaluate :

$$\int_{-4}^{0} |x + 2|\, dx$$

24. (a) If $y = (\sin^{-1} x)^2$, then find $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$.

OR

(b) If $y^x = x^y$, then find $\dfrac{dy}{dx}$.

25. Find the value of k so that the lines joining the points (1, – 1, 2) and (3, 4, k) is perpendicular to the line joining the points (0, 3, 2) and (3, 5, 6).

SECTION C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?

27. (a) If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$, then find $\frac{dy}{dx}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

28. (a) Find a matrix A such that

$$A\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

29. Find the particular solution of the differential equation :

$x \cos y \, dy = (x\, e^x \log x + e^x)\, dx$ given that $y = \frac{\pi}{2}$ when $x = 1$.

30. Show that the vectors $3\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} - 8\hat{k}$ and $4\hat{i} - 2\hat{j} - 7\hat{k}$ form the vertices of a right triangle.

31. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x - 3}{\sqrt{1 + 4x - 2x^2}} dx$$

SECTION D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Check whether the relation S in the set $\mathbb{R}$ of real numbers, defined as $S = \{(a, b) : a \leq b^2\}$ is reflexive, symmetric or transitive. Also, determine all $x \in R$ such that $(x, x) \in S$.

33. Solve the following linear programming problem graphically :

Minimise $Z = 6x + 7y$

subject to constraints

$$x + 2y \geq 240$$
$$3x + 4y \leq 620$$
$$2x + y \geq 180$$
$$x, y \geq 0.$$

34. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

35. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

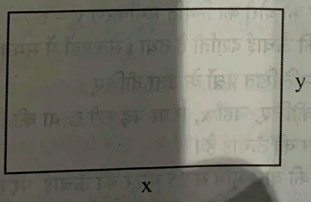
$\begin{array}{r} 90 \\ -12 \\ \hline 78 \end{array}$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.



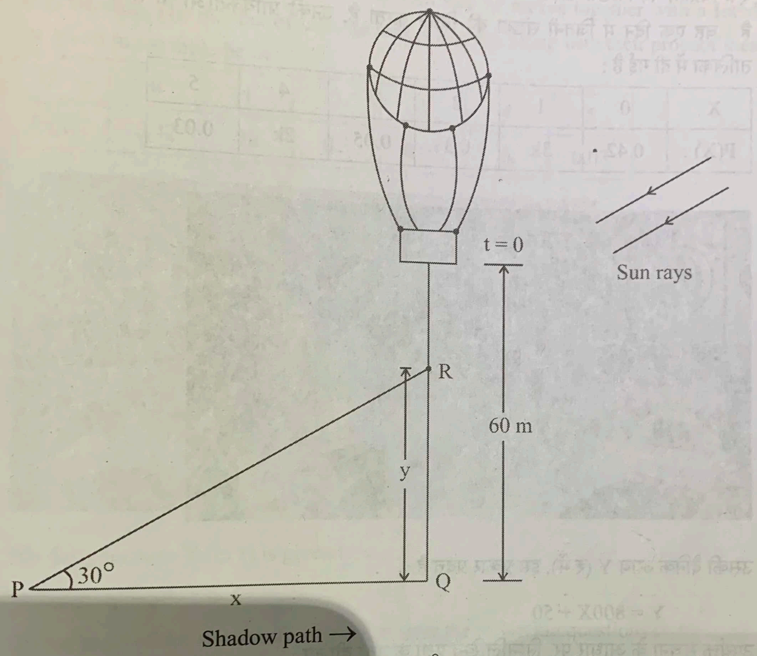
On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

Case Study – 2

37. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9\,t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Case Study – 3

38. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

On the basis of the above information, answer the following questions :

(i) Find the value of k.

(ii) Evaluate $P(X \geq 3)$.

(iii) (a) Calculate the expected weekly income of the salesman assuming he works five days per week.

OR

(iii) (b) Calculate the expected weekly income of the salesman assuming he works only for three days of the week.

2024 Annual

Series &RQPS

Set – 5

प्रश्न-पत्र कोड
Q.P. Code **65(B)**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then the value of x, for which A is an identity matrix, is

(A) $\frac{\pi}{2}$ (B) π

(C) 0 (D) $\frac{3\pi}{2}$

2. If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of 'a' and 'b' are :

(A) $a = 5, b = 3$
(B) $a = 5, b = -7$
(C) $a = -5, b = -7$
(D) $a = -5, b = 7$

3. If $\begin{vmatrix} x+2 & x-4 \\ x-2 & x+3 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$, then the value of x is :

(A) 1
(B) 2
(C) -2
(D) -1

4. If $\begin{bmatrix} 8 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X$, then matrix X is :

(A) $\begin{bmatrix} 3 & 7 \\ 2 & 0 \end{bmatrix}$
(B) $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$
(C) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$
(D) $\begin{bmatrix} 2 & 0 \\ -3 & 7 \end{bmatrix}$

5. The value of k, for which $f(x) = \begin{cases} \dfrac{\sqrt{3}\cos x + \sin x}{3x + \dfrac{\pi}{2}}, & x \neq -\dfrac{\pi}{3} \\ k, & x = -\dfrac{\pi}{3} \end{cases}$

is continuous at $x = -\dfrac{\pi}{3}$, is :

(A) $\dfrac{2}{3}$
(B) $-\dfrac{2}{3}$
(C) $\dfrac{3}{2}$
(D) 6

6. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \frac{2}{\sqrt{3}}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

7. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then the projection of $(\vec{c} - \vec{b})$ along $\vec{a}$ is :

(A) 15 (B) 5

(C) $\frac{2}{3}$ (D) 1

8. The angle between the lines

$$\frac{x+1}{2} = \frac{2-y}{-5} = \frac{z}{4} \text{ and } \frac{x-3}{1} = \frac{y-7}{2} = \frac{5-z}{3} \text{ is :}$$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

9. The Cartesian equations of a line are given as

$$6x - 2 = 3y + 1 = 2z - 2$$

The direction ratios of the line are :

(A) 2, – 1, 3 (B) 1, – 2, – 3

(C) 1, 2, 3 (D) 3, 1, 2

10. The solution set of the inequation $2x + 3y < 6$ is :

(A) open half-plane not containing origin

(B) whole xy-plane except the points lying on the line $2x + 3y = 6$

(C) open half-plane containing origin

(D) half-plane containing the origin and the points lying on the line $2x + 3y = 6$

11. The maximum value of the objective function $z = 3x + 5y$ subject to the constraints $x \geq 0$, $y \geq 0$ and $4x + 3y \leq 12$ is :

(A) 15 (B) 29

(C) 9 (D) 20

12. If the points A(3, – 2), B(k, 2) and C(8, 8) are collinear, then the value of k is :

(A) 2 (B) – 3

(C) 5 (D) – 4

13. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ is equal to :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) $-\frac{3}{2}$

14. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to :

(A) $\cot x + \tan x + c$ (B) $-\cot x + \tan x + c$

(C) $\cot x - \tan x + c$ (D) $-\cot x - \tan x + c$

15. The solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ is :

(A) $\log |1 + y| = x - \frac{x^2}{2} + c$ (B) $\log |1 + y| = -x + \frac{x^2}{2} + c$

(C) $e^y = x - \frac{x^2}{2} + c$ (D) $e^{(1+y)} = -x + \frac{x^2}{2} + c$

16. The degree of the differential equation

$$x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^5 = 0 \text{ is :}$$

(A) 2 (B) 3

(C) 4 (D) 5

17. The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$ is :

(A) $e^{\sec x}$ (B) $\sec x + \tan x$

(C) $\sec x$ (D) $\cos x$

18. The probabilities of A, B and C solving a problem are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. The probability that the problem is solved, is :

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$

(C) $\frac{1}{90}$ (D) $\frac{1}{3}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

Reason (R) : $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

20. *Assertion (A) :* If the side of a square is increasing at the rate of 0·2 cm/s, then the rate of increase of its perimeter is 0·8 cm/s.

Reason (R) : Perimeter of a square = 4 (side).

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

OR

(b) Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

22. (a) Differentiate $\cot^{-1}(\sqrt{1 + x^2} + x)$ w.r.t. x.

OR

(b) If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

23. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.

24. Show that of all rectangles inscribed in a given circle, the square has the maximum area.

25. Find :

$$\int \text{cosec}^3 (3x+1) \cot (3x+1)\, dx$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. If $x = a \cos \theta$ and $y = b \sin \theta$, then prove that $\dfrac{d^2y}{dx^2} = -\dfrac{b^4}{a^2y^3}$.

27. Find :

$$\int \frac{2x}{(x^2+1)(x^2+3)}\, dx$$

28. (a) Evaluate :

$$\int_{-6}^{6} |x+2|\, dx$$

OR

(b) Find :

$$\int \left(\frac{4-x}{x^5}\right) e^x\, dx$$

29. (a) Find the particular solution of the differential equation $2xy \frac{dy}{dx} = x^2 + 3y^2$, given that $y(1) = 0$.

OR

(b) Solve the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$.

30. The corner points of the feasible region determined by the system of linear constraints are A(0, 40), B(20, 40), C(60, 20) and D(60, 0). The objective function of the L.P.P. is $z = 4x + 3y$. Find the point of the feasible region at which the value of objective function is maximum and the point at which the value is minimum. Hence, find the maximum and the minimum values.

31. (a) A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under :

A : Getting a card of diamond

B : Getting a queen

Determine whether the events A and B are independent or not.

OR

(b) Find the probability distribution of the number of doublets in three throws of a pair of dice.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Let $A = \{x \mid x \in Z, 0 \leq x \leq 12\}$. Show that the relation $R = \{(a, b) : a, b \in A, (a - b)$ is divisible by $4\}$ is an equivalence relation. Find the set of elements related to 2.

OR

(b) Let $A = \mathbb{R} - \{4\}$ and $B = \mathbb{R} - \{1\}$ and let function $f : A \to B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall\, x \in A$. Show that f is one-one and onto.

33. Using matrices, solve the following system of linear equations :

$$3x + 4y + 2z = 8 \;;\; 2y - 3z = 3 \;;\; x - 2y + 6z = -2$$

34. Using integration, find the area of the region bounded by the curve $y = x^2$, $x = -1$, $x = 1$ and the x-axis.

35. (a) Write the vector equations of the following lines and hence find the shortest distance between them :

$$\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

OR

(b) Find the length and the coordinates of the foot of the perpendicular drawn from the point P(5, 9, 3) to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight.

Based on the above, answer the following questions :

(i) Find the rate of growth of the plant with respect to sunlight. *1*

(ii) What is the number of days it will take for the plant to grow to the maximum height ? *2*

(iii) What is the maximum height of the plant ? *1*

Case Study – 2

37. A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD, where the points are A(3, 4, 0), B(5, 3, 3), C(6, – 4, 1) and D(13, – 5, – 4).

Based on the above, answer the following questions :

(i) Write the direction ratios of vector $\overrightarrow{AB}$. *1*

(ii) Write a unit vector in the direction of $\overrightarrow{CD}$. *1*

(iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. *2*

OR

(iii) (b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. *2*

Case Study – 3

38. A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and player C can hit 2 times in 3 shots.

Based on the above, answer the following questions :

(i) Find the probability that all three players miss the target. *1*

(ii) Find the probability that all of them hit the target. *1*

(iii) (a) Find the probability that only one of them hits the target. *2*

OR

(iii) (b) Find the probability that exactly two of them hit the target. *2*

2024 Annual

Series PQ1RS/1

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/1/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. A function $f : R_+ \rightarrow R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

2. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

3. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1 \quad , & x = 0 \end{cases}$?

(A) f(x) is continuous and differentiable $\forall\, x \in \mathbb{R}$

(B) f(x) is continuous $\forall\, x \in \mathbb{R}$

(C) f(x) is continuous and differentiable $\forall\, x \in \mathbb{R} - \{0\}$

(D) f(x) is discontinuous at infinitely many points

4. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

(A) $f'(x) < 0, \forall\, x \in (a, b)$

(B) $f'(x) > 0, \forall\, x \in (a, b)$

(C) $f'(x) = 0, \forall\, x \in (a, b)$

(D) $f(x) > 0, \forall\, x \in (a, b)$

5. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(A) 7 (B) 6

(C) 8 (D) 18

6. $\int_a^b f(x)\, dx$ is equal to :

(A) $\int_a^b f(a - x)\, dx$ (B) $\int_a^b f(a + b - x)\, dx$

(C) $\int_a^b f(x - (a + b))\, dx$ (D) $\int_a^b f((a - x) + (b - x))\, dx$

7. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$.

Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$

(C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$

8. The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is :

(A) $\frac{1}{x^2 - 1}$ (B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$ (D) $\frac{1}{\sqrt{1 - x^2}}$

9. If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is :

(A) ± 1 (B) $\pm \sqrt{3}$

(C) ± 3 (D) $\pm \frac{1}{3}$

10. A linear programming problem deals with the optimization of a/an :

(A) logarithmic function (B) linear function

(C) quadratic function (D) exponential function

11. If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$

12. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :

(A) $2x^3$ (B) 2

(C) 0 (D) $2x^3 - 2$

13. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :

(A) 1 (B) -1

(C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$

14. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are :

(A) 1, 2 (B) 2, 3

(C) 2, 1 (D) 2, 6

15. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

16. The distance of point P(a, b, c) from y-axis is :

(A) b (B) b^2

(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

17. The number of corner points of the feasible region determined by constraints $x \geq 0$, $y \geq 0$, $x + y \geq 4$ is :

(A) 0 (B) 1

(C) 2 (D) 3

18. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :

(A) AB = O (B) AB = – BA

(C) BA = O (D) AB = BA

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$,

$|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

20. *Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Check whether the function $f(x) = x^2 |x|$ is differentiable at $x = 0$ or not.

OR

(b) If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\,\frac{dy}{dx} = \frac{1 + y^4}{4y}$.

22. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

23. (a) Find :

$$\int x\sqrt{1 + 2x}\, dx$$

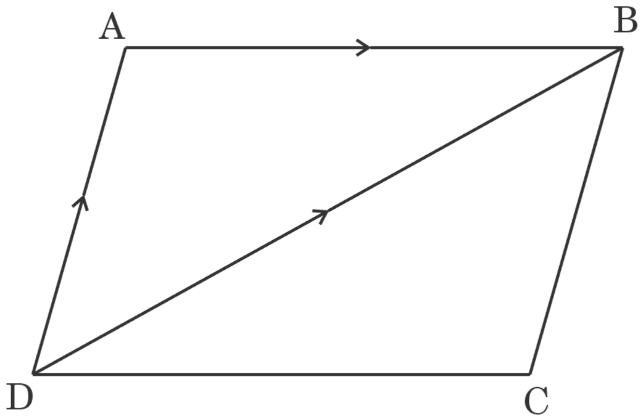
OR

(b) Evaluate :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

24. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\ |\vec{a}|$.

25. In the given figure, ABCD is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\vec{AD}$ and hence find the area of parallelogram ABCD.



SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

27. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a\,(x-y)$, prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\dfrac{dy}{dx}$.

28. (a) Find :

$$\int \frac{x^2}{(x^2+4)\,(x^2+9)}\,dx$$

OR

(b) Evaluate :

$$\int_1^3 \left(|x-1|+|x-2|+|x-3|\right)\,dx$$

29. Find the particular solution of the differential equation given by $x^2\dfrac{dy}{dx} - xy = x^2\cos^2\left(\dfrac{y}{2x}\right)$, given that when $x = 1$, $y = \dfrac{\pi}{2}$.

30. Solve the following linear programming problem graphically :

Maximise $z = 500x + 300y$,

subject to constraints

$x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x \geq 0,\ y \geq 0$

31. E and F are two independent events such that $P(\bar{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

33. (a) Evaluate :

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x)\, dx$$

34. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

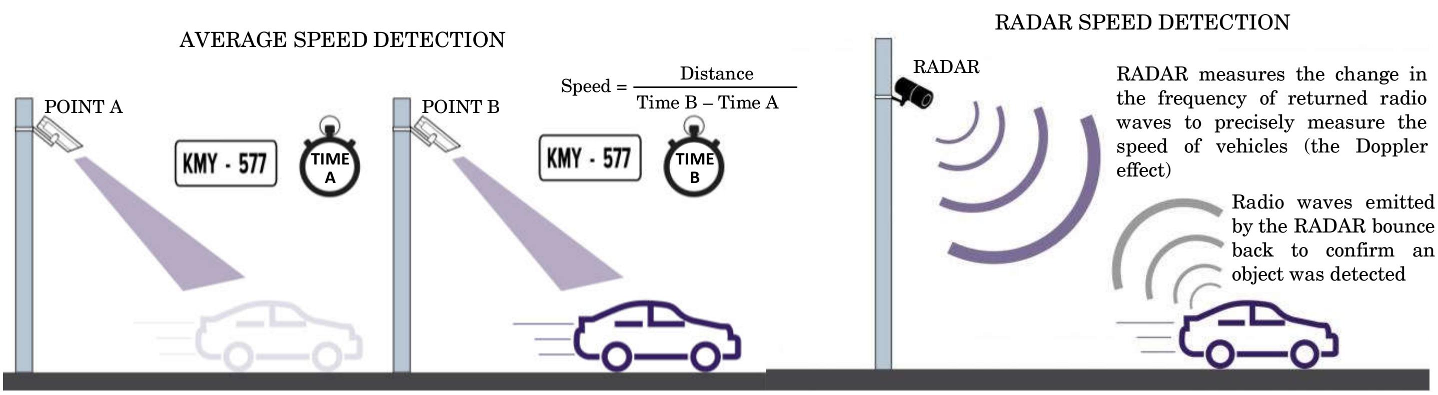
35. The image of point P(x, y, z) with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is P′ (1, 0, 7). Find the coordinates of point P.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. *1*

(ii) Find $\frac{d\theta}{dx}$. *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. *2*

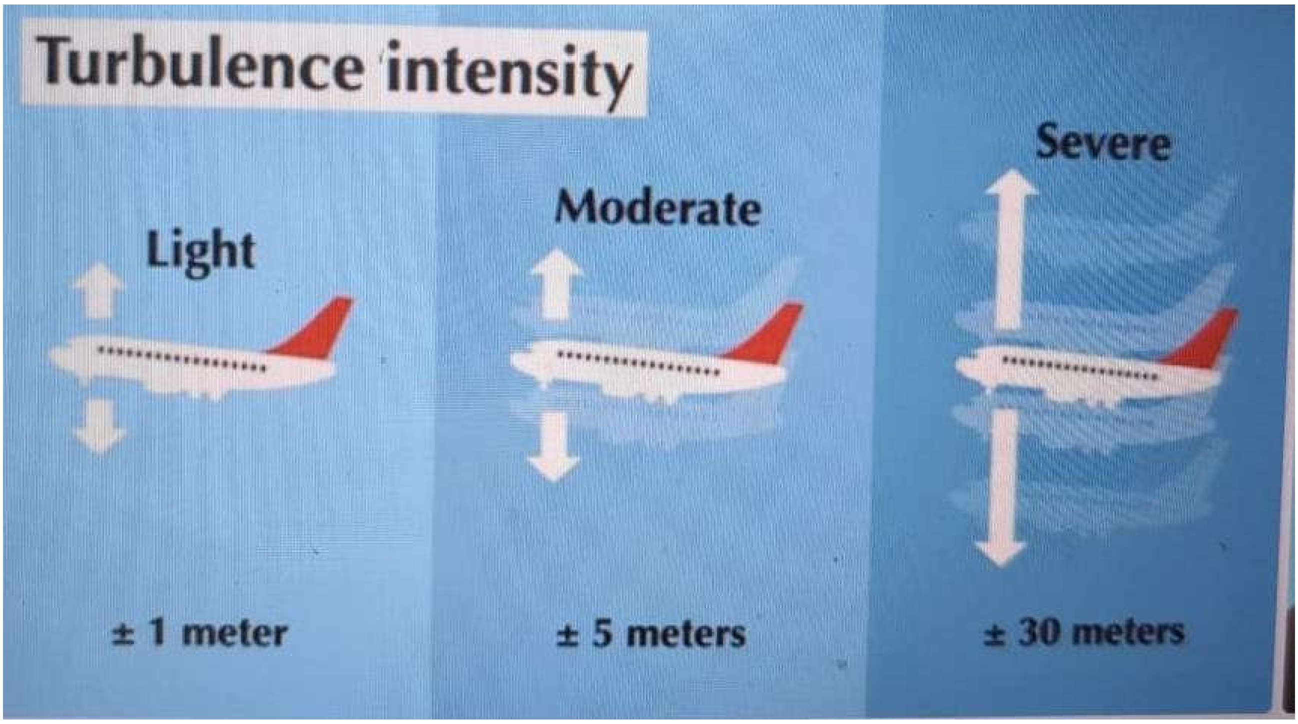
OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. *2*

Case Study – 2

37. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

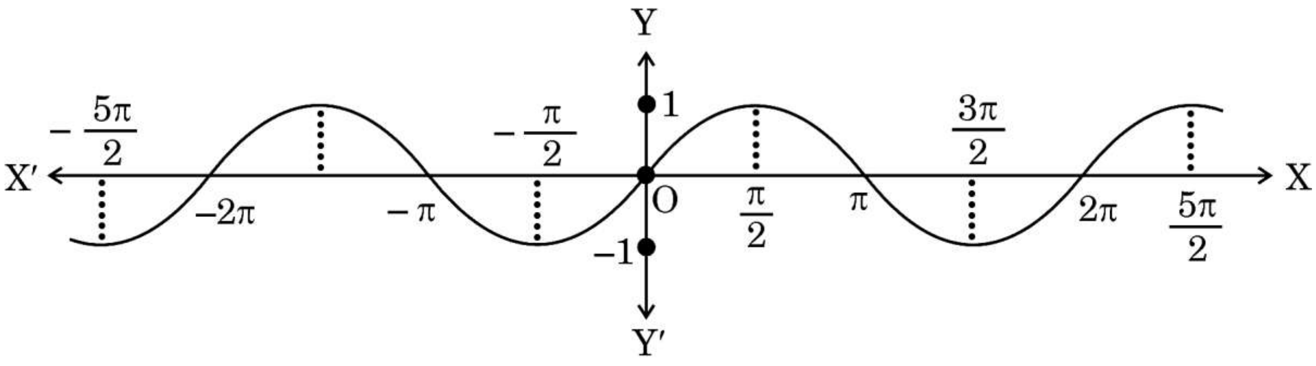
(i) Find the probability that an airplane reached its destination late. *2*

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. *2*

Case Study – 3

38. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



$y = \sin x$

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

2024 Annual

Series PQ1RS/1

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/1/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
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- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$.

Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm \frac{3}{5}$ (B) $\pm \frac{3}{4}$

(C) $\pm \frac{4}{5}$ (D) $\pm \frac{4}{3}$

2. The integrating factor of the differential equation $x \frac{dy}{dx} - y = x^4 - 3x$ is :

(A) x (B) $-x$

(C) x^{-1} (D) $\log(x^{-1})$

3. If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is :

(A) ± 1 (B) $\pm \sqrt{3}$

(C) ± 3 (D) $\pm \frac{1}{3}$

4. A linear programming problem deals with the optimization of a/an :

(A) logarithmic function (B) linear function

(C) quadratic function (D) exponential function

5. If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$

6. If a_{ij} and A_{ij} represent the $(ij)^{th}$ element and its cofactor of $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ respectively, then the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is :

(A) 0 (B) -28

(C) 114 (D) -114

7. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :

(A) 1 (B) -1

(C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$

8. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are :

(A) 1, 2 (B) 2, 3

(C) 2, 1 (D) 2, 6

9. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

10. The distance of point P(a, b, c) from y-axis is :

(A) b (B) b^2

(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

11. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :

(A) 0 (B) 1

(C) 2 (D) 3

12. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then :

(A) AB = O (B) AB = – BA

(C) BA = O (D) AB = BA

13. A relation R defined on set $A = \{x : x \in Z \text{ and } 0 \leq x \leq 10\}$ as $R = \{(x, y) : x = y\}$ is given to be an equivalence relation. The number of equivalence classes is :

(A) 1 (B) 2

(C) 10 (D) 11

14. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

15. The number of points, where f(x) = [x], 0 < x < 3 ([·] denotes greatest integer function) is not differentiable is :

(A) 1 (B) 2

(C) 3 (D) 4

16. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

(A) $f'(x) < 0, \forall x \in (a, b)$

(B) $f'(x) > 0, \forall x \in (a, b)$

(C) $f'(x) = 0, \forall x \in (a, b)$

(D) $f(x) > 0, \forall x \in (a, b)$

17. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(A) 7 (B) 6

(C) 8 (D) 18

18. If $\int_0^a \frac{1}{4+x^2}\, dx = \frac{\pi}{6}$, then the value of 'a' is :

(A) $\frac{\sqrt{3}}{2}$ (B) $2\sqrt{3}$

(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

20. *Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find :

$$\int x\sqrt{1+2x}\; dx$$

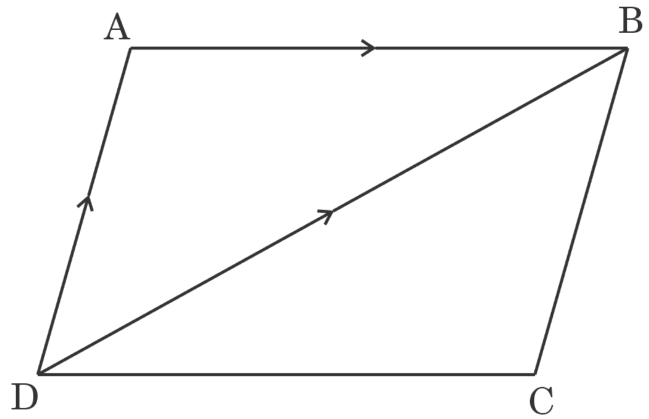
OR

(b) Evaluate :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

22. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\, |\vec{a}|$.

23. In the given figure, ABCD is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\vec{AD}$ and hence find the area of parallelogram ABCD.



24. (a) If $y = \sqrt{\cos x + y}$, prove that $\dfrac{dy}{dx} = \dfrac{\sin x}{1 - 2y}$.

OR

(b) Show that the function $f(x) = |x|^3$ is differentiable at all points of its domain.

25. Find the absolute maximum and minimum values of the function $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [0, 1]$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find :

$$\int \frac{x^2}{(x^2 + 4)(x^2 + 9)}\, dx$$

OR

(b) Evaluate :

$$\int_1^3 \left(|x - 1| + |x - 2| + |x - 3|\right) dx$$

27. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$.

28. Solve the following linear programming problem graphically :

Maximise $z = 5x + 4y$

subject to the constraints

$$x + 2y \geq 4$$
$$3x + y \leq 6$$
$$x + y \leq 4$$
$$x, y \geq 0$$

29. E and F are two independent events such that $P(\bar{E}) = 0 \cdot 6$ and $P(E \cup F) = 0 \cdot 6$. Find P(F) and $P(\bar{E} \cup \bar{F})$.

30. (a) A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

31. (a) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a\,(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. (a) Evaluate :

$$\int_{0}^{\pi/2} e^{x}\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

OR

(b) Find :

$$\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$$

33. Using integration, find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$, included between the lines $x=-2$ and $x=2$.

34. Equations of sides of a parallelogram ABCD are as follows :

AB : $\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-1}{2}$

BC : $\frac{x-1}{3}=\frac{y+2}{-5}=\frac{z-5}{3}$

CD : $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$

DA : $\frac{x-2}{3}=\frac{y+3}{-5}=\frac{z-4}{3}$

Find the equation of diagonal BD.

35. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

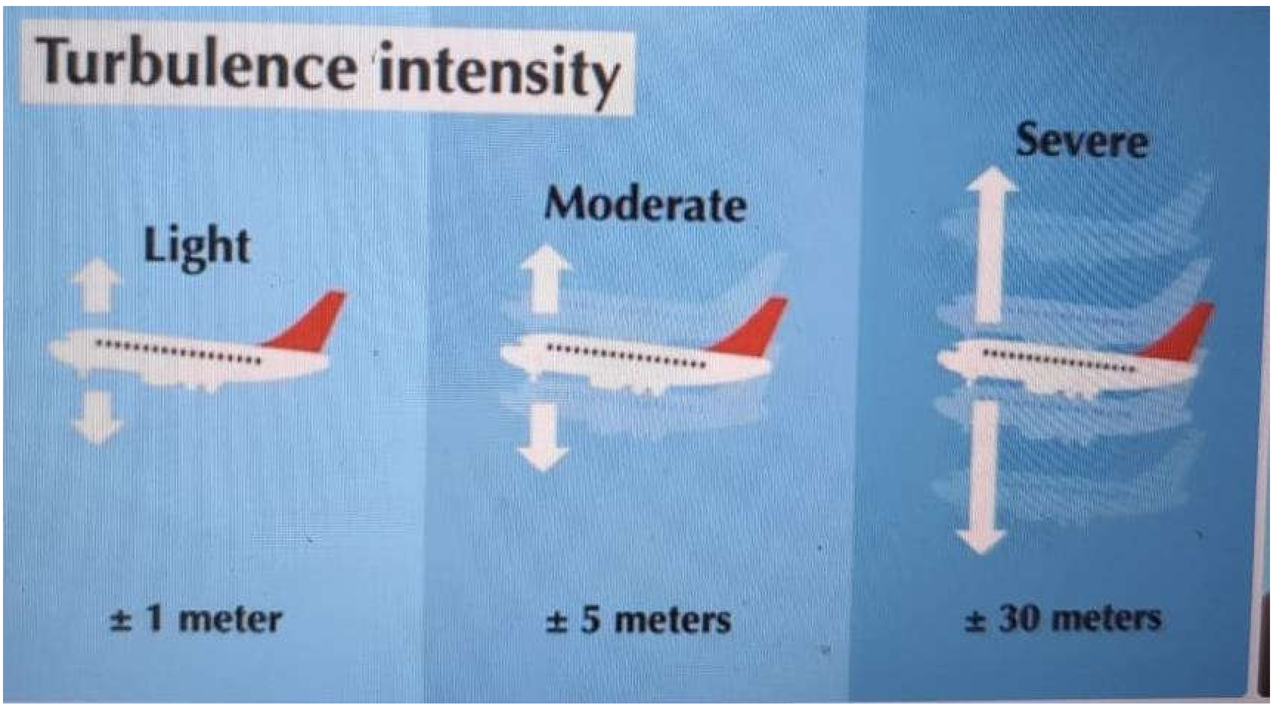
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

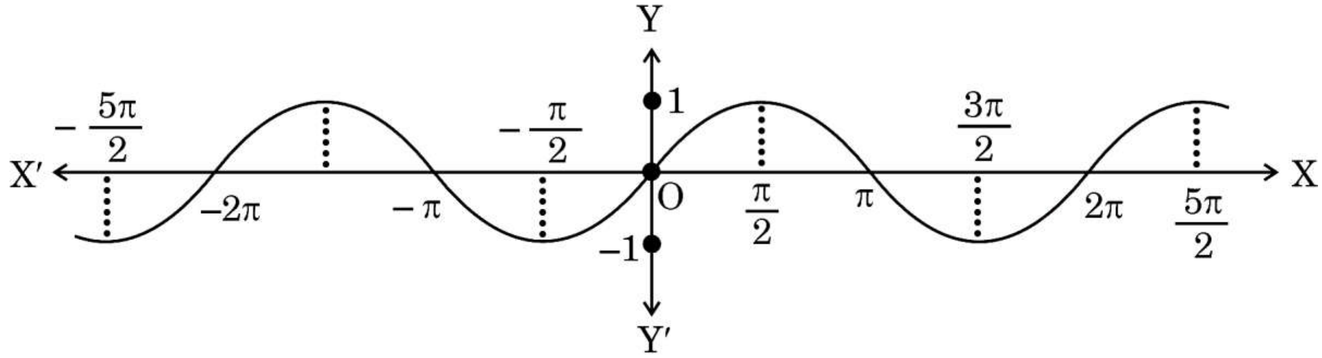
(i) Find the probability that an airplane reached its destination late. *2*

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. *2*

Case Study – 2

37. If a function $f : X \rightarrow Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \rightarrow R$ is neither one-one nor onto. The following graph shows the sine function.



$y = \sin x$

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

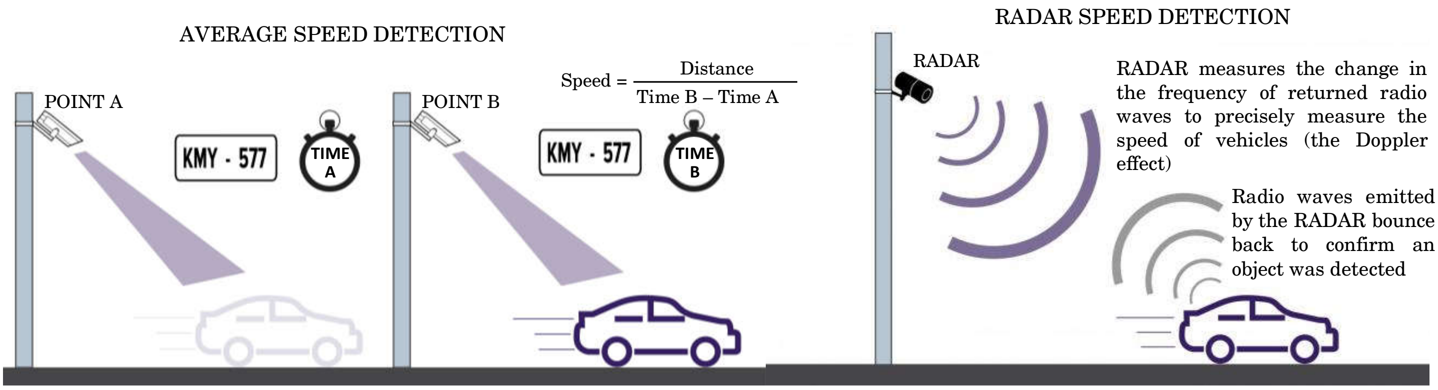
(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

Case Study – 3

38. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. *1*

(ii) Find $\frac{d\theta}{dx}$. *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. *2*

OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. *2*

2024 Annual

Series PQ1RS/1

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/1/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
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- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

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General Instructions :

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(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $x = at$, $y = \frac{a}{t}$, then $\frac{dy}{dx}$ is :

(A) t^2 (B) $-t^2$

(C) $\frac{1}{t^2}$ (D) $-\frac{1}{t^2}$

2. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{\log y}$ is :

(A) $\log y = x + c$ (B) $y \log y - y = x + c$

(C) $\log y - y = x + c$ (D) $y \log y + y = x + c$

3. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$
~~~

4. The distance of point P(a, b, c) from y-axis is :

(A) b (B) b^2

(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

5. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :

(A) 0 (B) 1

(C) 2 (D) 3

6. If matrices A and B are of order 1×3 and 3×1 respectively, then the order of $A'B'$ is :

(A) 1×1 (B) 3×1

(C) 1×3 (D) 3×3

7. A relation R defined on a set of human beings as

$$R = \{(x, y) : x \text{ is 5 cm shorter than } y\}$$

is :

(A) reflexive only

(B) reflexive and transitive

(C) symmetric and transitive

(D) neither transitive, nor symmetric, nor reflexive

8. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

9. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1 , & x = 0 \end{cases}$?

(A) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$

(B) f(x) is continuous $\forall x \in \mathbb{R}$

(C) f(x) is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$

(D) f(x) is discontinuous at infinitely many points

10. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

(A) $f'(x) < 0, \forall x \in (a, b)$

(B) $f'(x) > 0, \forall x \in (a, b)$

(C) $f'(x) = 0, \forall x \in (a, b)$

(D) $f(x) > 0, \forall x \in (a, b)$

11. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(A) 7 (B) 6

(C) 8 (D) 18

12. If f(x) is an odd function, then $\int_{-\pi/2}^{\pi/2} f(x) \cos^3 x \, dx$ equals :

(A) $2\int_{0}^{\pi/2} f(x) \cos^3 x \, dx$ (B) 0

(C) $2\int_{0}^{\pi/2} f(x) \, dx$ (D) $2\int_{0}^{\pi/2} \cos^3 x \, dx$

13. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$

(C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$

14. The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is :

(A) $\frac{1}{x^2 - 1}$ (B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$ (D) $\frac{1}{\sqrt{1 - x^2}}$

15. If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is :

(A) ± 1 (B) $\pm \sqrt{3}$

(C) ± 3 (D) $\pm \frac{1}{3}$

16. A linear programming problem deals with the optimization of a/an :

(A) logarithmic function (B) linear function

(C) quadratic function (D) exponential function

17. If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$

18. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :

(A) $2x^3$ (B) 2

(C) 0 (D) $2x^3 - 2$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

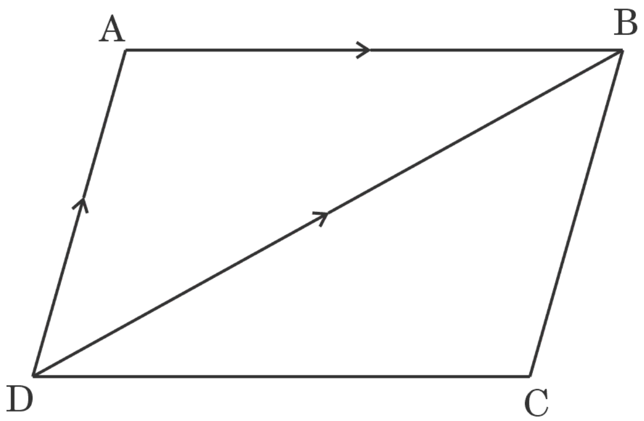
20. *Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



22. (a) Check the differentiability of function f(x) = [x] at x = – 3, where [•] denotes greatest integer function.

OR

(b) If $x^{1/3} + y^{1/3} = 1$, find $\dfrac{dy}{dx}$ at the point $\left(\dfrac{1}{8}, \dfrac{1}{8}\right)$.

23. Find local maximum value and local minimum value (whichever exists) for the function $f(x) = 4x^2 + \dfrac{1}{x}$ $(x \neq 0)$.

24. (a) Find :

$$\int x\sqrt{1+2x}\ dx$$

OR

(b) Evaluate :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$$

25. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\ |\vec{a}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically :

Minimise $z = 5x - 2y$

subject to the constraints

$$x + 2y \leq 120$$
$$x + y \geq 60$$
$$x - 2y \geq 0$$
$$x, y \geq 0$$

27. E and F are two independent events such that $P(\overline{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find P(F) and $P(\overline{E} \cup \overline{F})$.

28. (a) A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

29. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a\,(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

30. (a) Find :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1|+|x-2|+|x-3|)\, dx$$

31. Solve the following differential equation :

$$(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. Find the equation of a line l_2 which is the mirror image of the line l_1 with respect to line $l : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line l_1 passes through the point P(1, 6, 3) and parallel to line l.

33. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

34. (a) Find :

$$\int \frac{(3\cos x - 2)\sin x}{5 - \sin^2 x - 4\cos x}\, dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^3 + |x| + 1}{x^2 + 4|x| + 4}\, dx$$

35. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

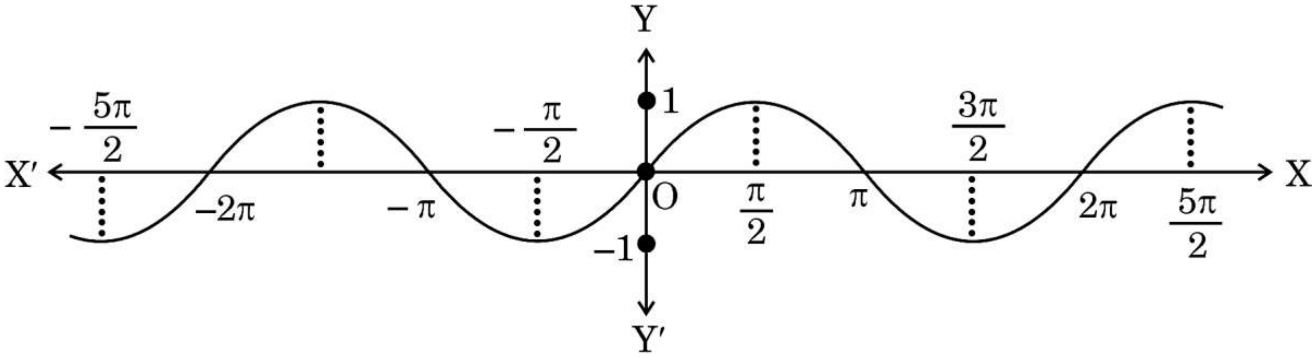
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



$y = \sin x$

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

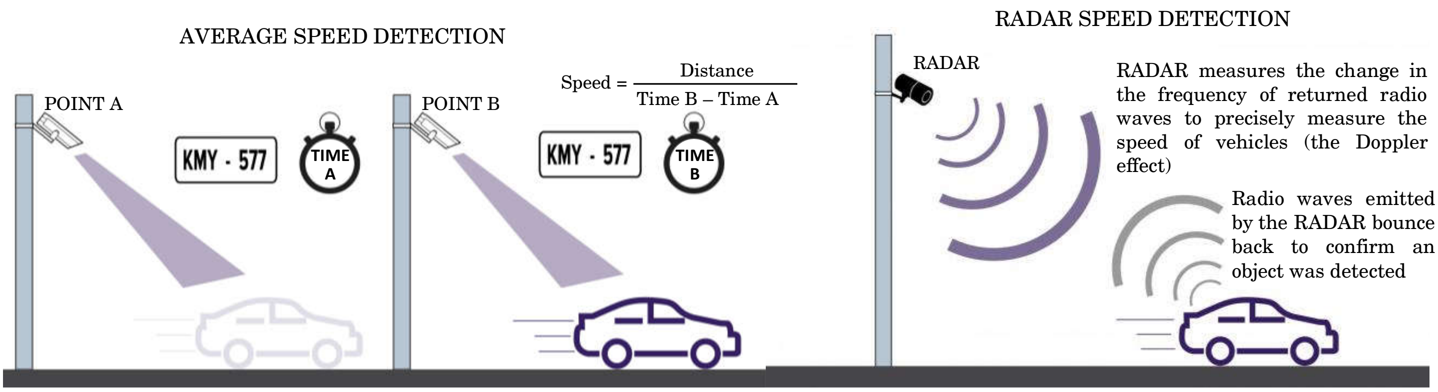
(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

Case Study – 2

37. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. *1*

(ii) Find $\frac{d\theta}{dx}$. *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. *2*

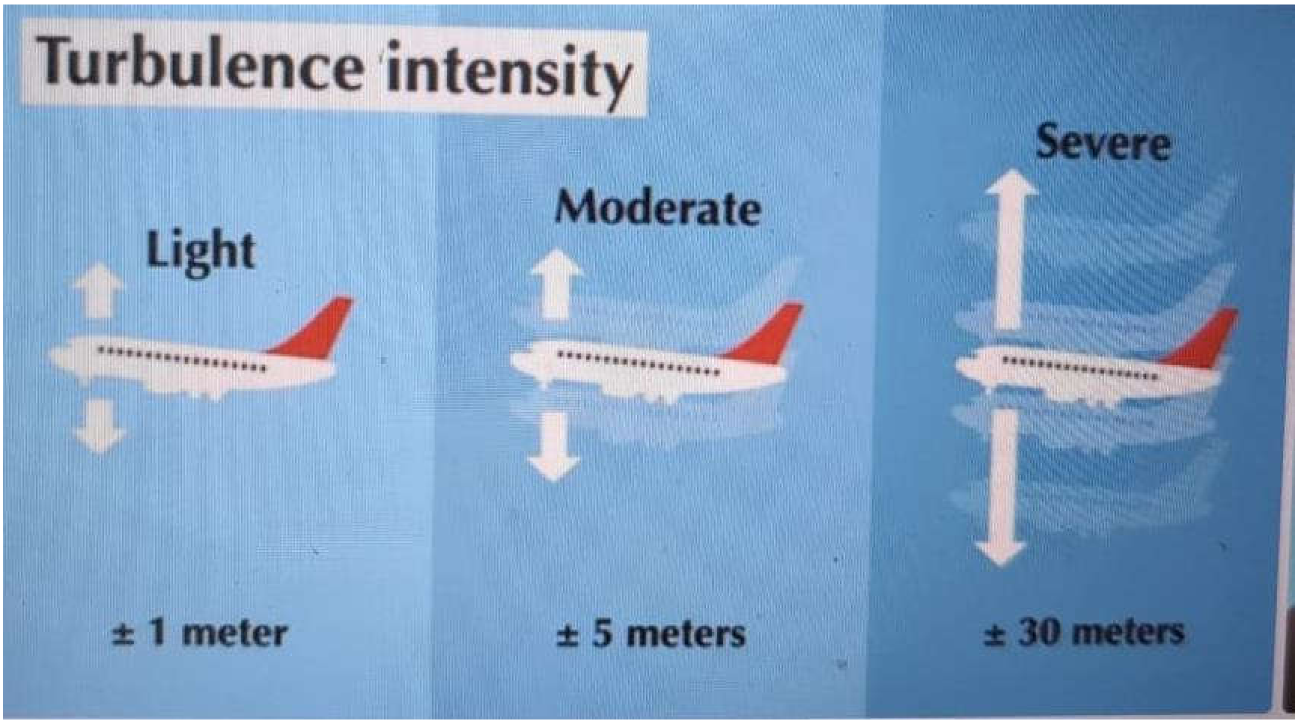
OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. *2*

Case Study – 3

38. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

(i) Find the probability that an airplane reached its destination late. *2*

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. *2*

2024 Annual

Series PQ2RS/2

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/2/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

2. Let $f : R_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(A) one-one

(B) onto

(C) bijective

(D) neither one-one nor onto

3. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

4. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3<x<3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

5. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

6. $\int_{0}^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to :

(A) π

(B) Zero (0)

(C) $\int_{0}^{\pi/2} \frac{2 \sin x}{1 + \sin x \cos x} dx$

(D) $\frac{\pi^2}{4}$

7. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is :

(A) $\cos x - \sin \left(\frac{y}{x}\right)$

(B) $\frac{y}{x}$

(C) $\frac{x^2 + y^2}{xy}$

(D) $\cos^2 \left(\frac{x}{y}\right)$

8. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}| \, |\vec{b}|$

(B) $\vec{a} \cdot \vec{b} = |\vec{a}| \, |\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}| \, |\vec{b}|$

(D) $\vec{a} \cdot \vec{b} < |\vec{a}| \, |\vec{b}|$

9. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0)

(B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$

(D) (0, 0, 0)

10. The common region determined by all the constraints of a linear programming problem is called :

(A) an unbounded region

(B) an optimal region

(C) a bounded region

(D) a feasible region

11. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

12. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is ***false*** ?

(A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

13. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

(A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$

(C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$

14. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is :

(A) 1 (B) 2

(C) 3 (D) not defined

15. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

16. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

17. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :

(A) 1 (B) 2

(C) 0 (D) – 2

18. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if P′ = – P.

20. *Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

22. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(b) If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1 + x^2}\,\frac{dy}{dx} - x = 0$.

23. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

24. Find :

$$\int \frac{e^{4x} - 1}{e^{4x} + 1}\,dx$$

25. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

OR

(b) Show that :

$$\frac{d}{dx}(|x|) = \frac{x}{|x|},\ x \neq 0$$

27. (a) Evaluate :

$$\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$$

OR

(b) Find :

$$\int \frac{1}{x\,[(\log x)^2 - 3 \log x - 4]}\, dx$$

28. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation :

$$y\, dx = (x + 2y^2)\, dy$$

29. The position vectors of vertices of Δ ABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of Δ ABC.

30. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

31. Find :

$$\int x^2 \cdot \sin^{-1}(x^{3/2})\, dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Show that a function $f : R \rightarrow R$ defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \rightarrow A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :

$$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

33. Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$.

34. (a) Solve the following system of equations, using matrices :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

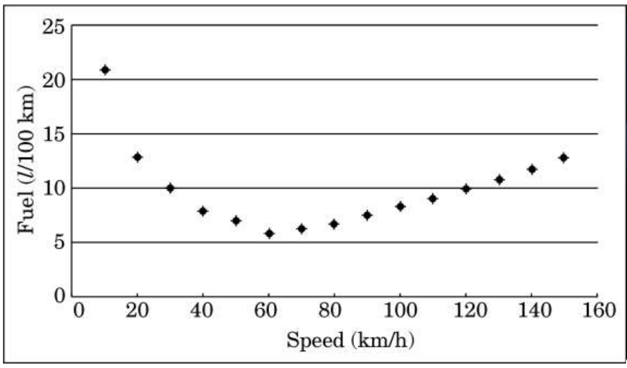
35. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\frac{dF}{dV}$. *1*

(iii) (a) Find the speed V for which fuel consumption F is minimum. *2*

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$. *2*

Case Study – 2

37. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

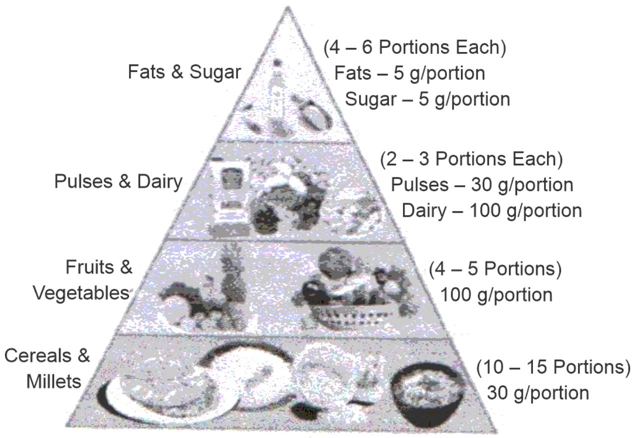


Figure-1

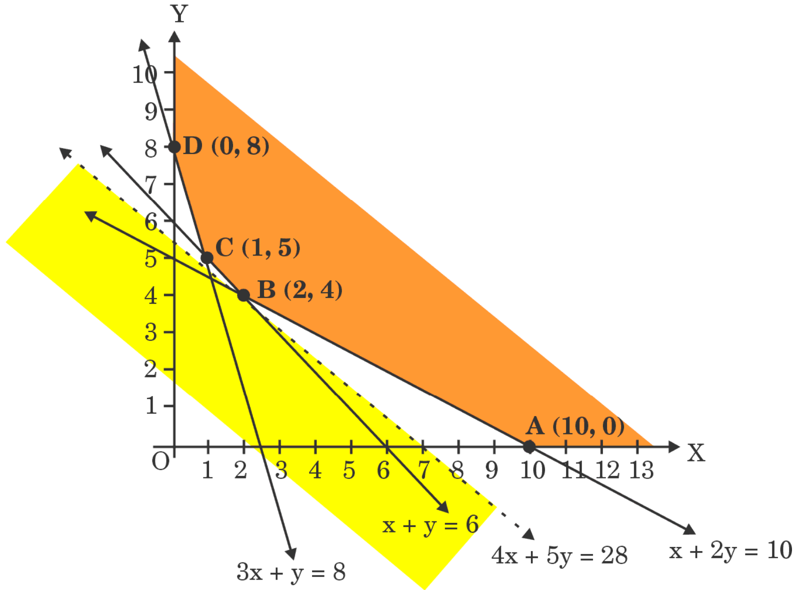


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

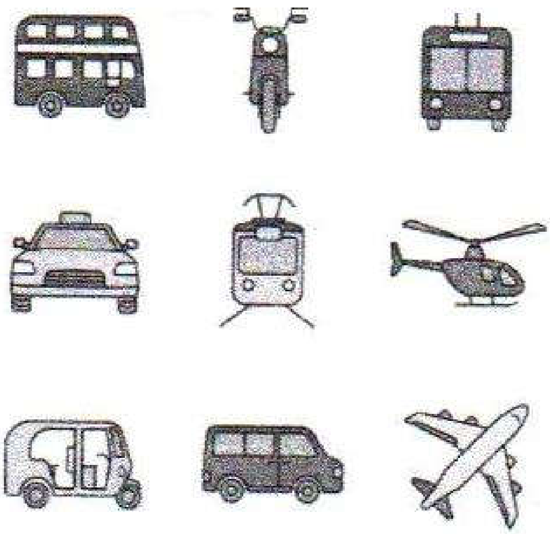
On the basis of the above information, answer the following questions :

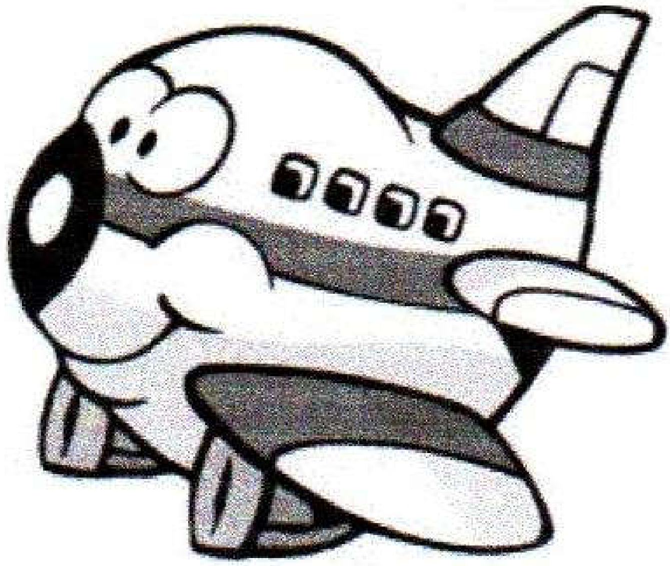
(i) Identify and write all the constraints which determine the given feasible region in Figure-2. *2*

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. *2*

Case Study – 3

38. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash. *1*

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$. *1*

(iii) (a) Find P(A). *2*

OR

(iii) (b) Find $P(E_2 \mid A)$. *2*

2024 Annual

Series PQ2RS/2

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/2/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is :

(A) 0 (B) 1

(C) 2 (D) infinitely many

2. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

3. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

4. The common region determined by all the constraints of a linear programming problem is called :

(A) an unbounded region (B) an optimal region

(C) a bounded region (D) a feasible region

5. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

(A) $P(S \cap E)$ (B) P(E)

(C) 1 (D) 0

6. The number of all scalar matrices of order 3, with each entry – 1, 0 or 1, is :

(A) 1 (B) 3

(C) 2 (D) 3^9

7. $\frac{d}{dx}[\cos(\log x + e^x)]$ at x = 1 is :

(A) – sin e (B) sin e

(C) – (1 + e) sin e (D) (1 + e) sin e

8. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is :

(A) 1 (B) 2

(C) 3 (D) not defined

9. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\dfrac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\dfrac{\hat{i} + \hat{k}}{\sqrt{2}}$

10. Direction ratios of a vector parallel to line $\dfrac{x-1}{2} = -y = \dfrac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

11. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :

(A) 1 (B) 2

(C) 0 (D) – 2

12. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

13. If the sum of all the elements of a 3 × 3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

14. Which of the following statements is ***not*** true about equivalence classes A_i (i = 1, 2, ... n) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^{n} A_i = A$

(B) $A_i \cap A_j \neq \phi,\ i \neq j$

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

15. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

16. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3<x<3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

17. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

18. If $\int_{0}^{2} 2\,e^{2x}\,dx = \int_{0}^{a} e^{x}\,dx$, the value of 'a' is :

(A) 1 (B) 2

(C) 4 (D) $\frac{1}{2}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

20. *Assertion (A) :* For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

22. Evaluate :

$$\int_0^{a^3} \frac{x^2}{x^6 + a^6} \, dx$$

23. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

24. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

25. (a) Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

OR

(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation :

$$y\,dx = (x + 2y^2)\,dy$$

27. If vectors $\vec{a}$, $\vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.

28. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

29. Find :

$$\int x^2 \cdot \sin^{-1}(x^{3/2})\,dx$$

30. (a) If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

OR

(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.

31. (a) Evaluate :

$$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}}\,dx$$

OR

(b) Find :

$$\int \frac{1}{x\,[(\log x)^2 - 3\log x - 4]}\,dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the value of p for which the lines

$$\vec{r} = \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k} \text{ and}$$

$$\vec{r} = \hat{i} - 3\mu\hat{j} + (p\mu + 7)\hat{k}$$

are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

33. (a) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$.

Also, find $|(AB)^{-1}|$.

OR

(b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations :

$$x + y + z = 1$$
$$2x + 3y + 2z = 2$$
$$x + y + 2z = 4$$

34. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

35. (a) Show that a function $f : R \to R$ defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :

$$(a, b) \, R \, (c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

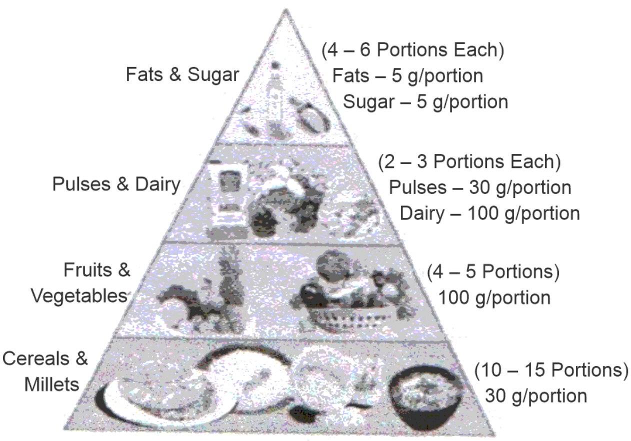


Figure-1

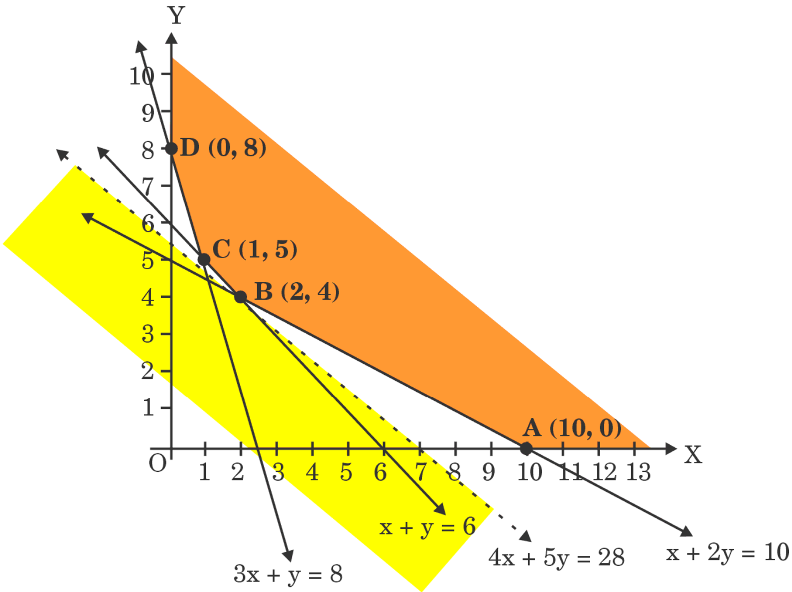


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

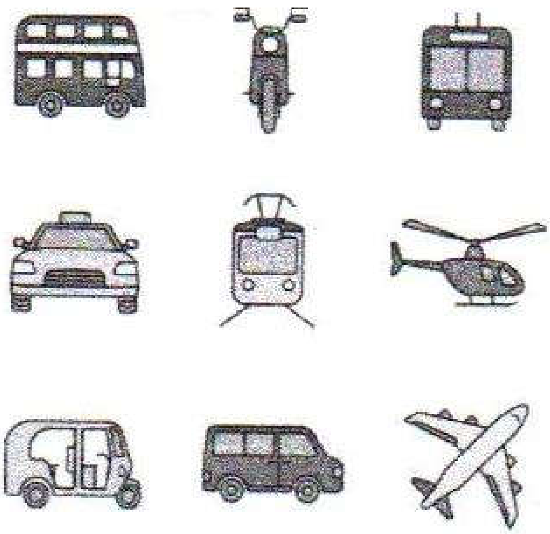
On the basis of the above information, answer the following questions :

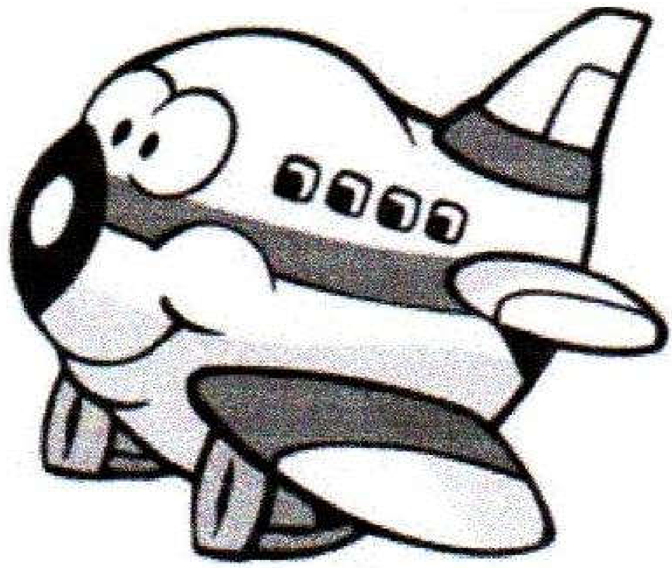
(i) Identify and write all the constraints which determine the given feasible region in Figure-2. *2*

(ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. *2*

Case Study – 2

37. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash. *1*

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$. *1*

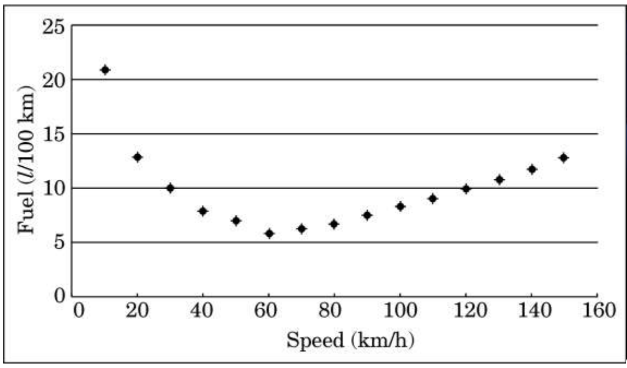
(iii) (a) Find P(A). *2*

OR

(iii) (b) Find $P(E_2 \mid A)$. *2*

Case Study – 3

38. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \dfrac{V^2}{500} - \dfrac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\dfrac{dF}{dV}$. *1*

(iii) (a) Find the speed V for which fuel consumption F is minimum. *2*

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\dfrac{dF}{dV} = -0{\cdot}01$. *2*

2024 Annual

Series PQ2RS/2

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/2/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

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(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $y = \cos^{-1}(e^x)$, then $\dfrac{dy}{dx}$ is :

(A) $\dfrac{1}{\sqrt{e^{-2x}+1}}$ (B) $-\dfrac{1}{\sqrt{e^{-2x}+1}}$

(C) $\dfrac{1}{\sqrt{e^{-2x}-1}}$ (D) $-\dfrac{1}{\sqrt{e^{-2x}-1}}$

2. The degree and order of differential equation $y''^2 + \log(y') = x^5$ respectively are :

(A) not defined, 5 (B) not defined, 2

(C) 5, not defined (D) 2, 2

3. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

4. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

5. If for the matrix $A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}\,I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is :

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

6. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

7. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

8. Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(A) one-one

(B) onto

(C) bijective

(D) neither one-one nor onto

9. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

10. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3<x<3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

11. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

12. Anti-derivative of $\sqrt{1 + \sin 2x}$, $x \in \left[0, \frac{\pi}{4}\right]$ is :

(A) $\cos x + \sin x$ (B) $-\cos x + \sin x$

(C) $\cos x - \sin x$ (D) $-\cos x - \sin x$

13. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$

(C) $\frac{x^2 + y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

14. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}|\,|\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}|\,|\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}|\,|\vec{b}|$

15. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

16. The common region determined by all the constraints of a linear programming problem is called :

(A) an unbounded region (B) an optimal region

(C) a bounded region (D) a feasible region

17. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

18. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is ***false*** ?

(A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

20. *Assertion (A) :* $(\vec{b} \cdot \vec{c})\,\vec{a}$ is a scalar quantity.

Reason (R) : Dot product of two vectors is a scalar quantity.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Determine whether the function $f(x) = x^2 - 6x + 3$ is increasing or decreasing in [4, 6].

22. (a) Find the value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(\tan\frac{4\pi}{3}\right)$.

OR

(b) Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range.

23. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(b) If $y = \operatorname{cosec}(\cot^{-1}x)$, then prove that $\sqrt{1+x^2}\,\frac{dy}{dx} - x = 0$.

24. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

25. Find :

$$\int \frac{e^{4x} - 1}{e^{4x} + 1}\, dx$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

27. Find :

$$\int x^2 \log (x^2 - 1)\, dx$$

28. (a) If $y = (\log x)^2$, prove that $x^2y'' + xy' = 2$.

OR

(b) If $y = \sin (\tan^{-1} e^x)$, then find $\frac{dy}{dx}$ at $x = 0$.

29. (a) Evaluate :

$$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}}\, dx$$

OR

(b) Find :

$$\int \frac{1}{x [(\log x)^2 - 3 \log x - 4]}\, dx$$

30. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation :

$$y\, dx = (x + 2y^2)\, dy$$

31. The position vectors of vertices of Δ ABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of Δ ABC.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

33. (a) Show that a function $f : R \rightarrow R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which $f(x) = 3$.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as $(a, b)\ R\ (c, d) \Leftrightarrow \frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation.

34. The vertices of Δ ABC are A(1, 1, 0), B(1, 2, 1) and C(–2, 2, –1). Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid.

35. (a) Solve the following system of equations, using matrices :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

OR

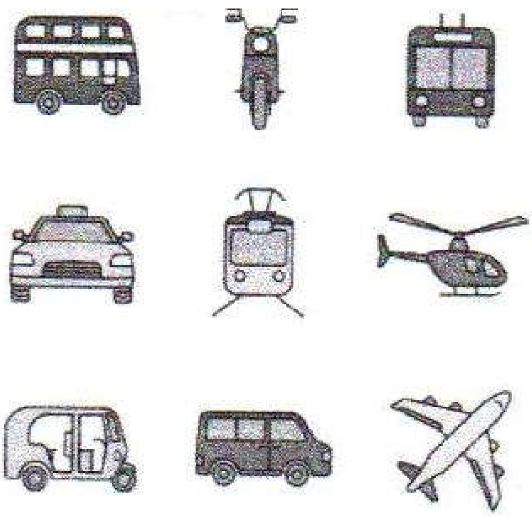
(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

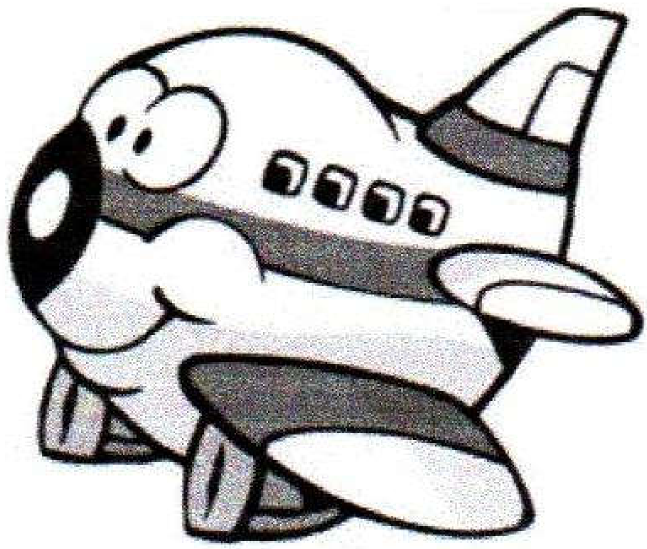
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash. *1*

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$. *1*

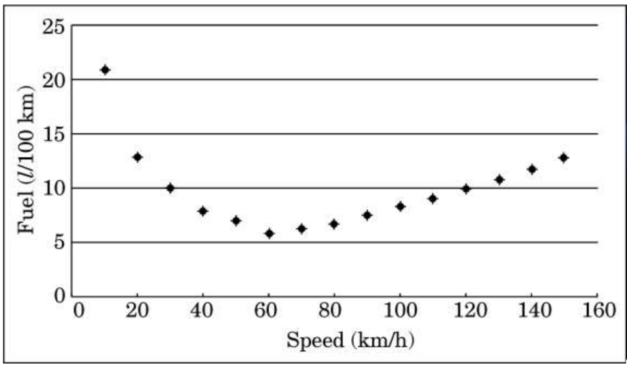
(iii) (a) Find P(A). *2*

OR

(iii) (b) Find $P(E_2 \mid A)$. *2*

Case Study – 2

37. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\frac{dF}{dV}$. *1*

(iii) (a) Find the speed V for which fuel consumption F is minimum. *2*

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$. *2*

Case Study – 3

38. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

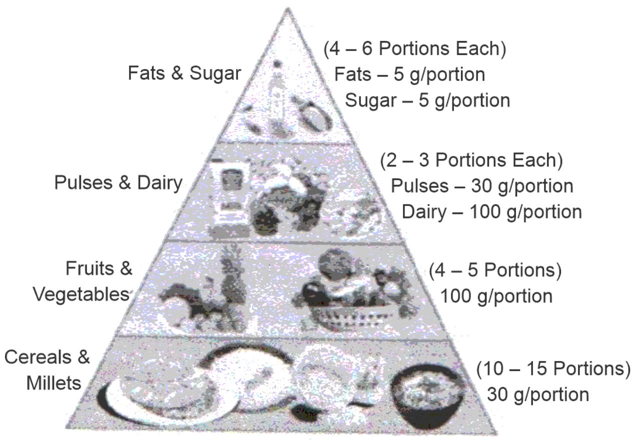


Figure-1

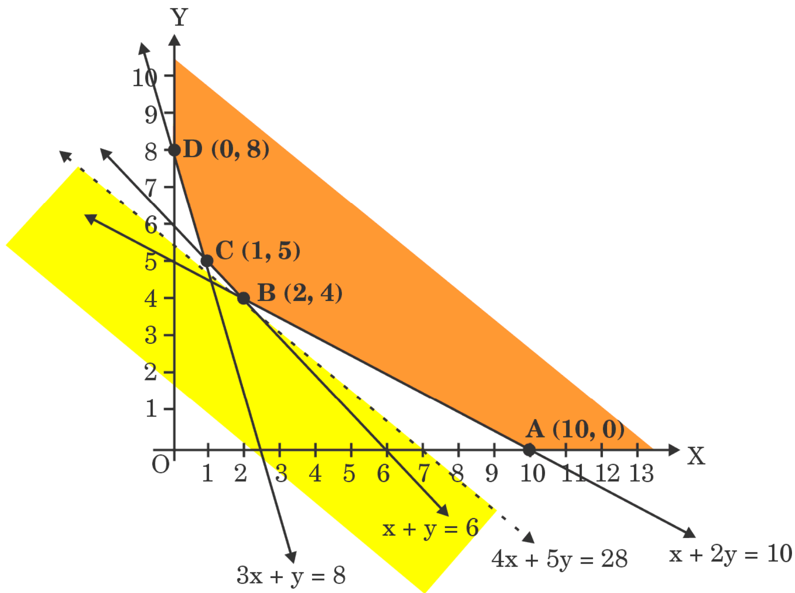


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions :

(i) Identify and write all the constraints which determine the given feasible region in Figure-2. *2*

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. *2*

2024 Annual

Series PQ3RS/3

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/3/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\, i, j$

(C) $a_{ij} = 0, \forall\, i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

2. Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(A) one-one but not onto (B) onto but not one-one

(C) both one-one and onto (D) neither one-one nor onto

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, (a + b + c + d) is equal to :

(A) 2a (B) 2b

(C) 2c (D) 0

4. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at $x = 1$ only (B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$ (D) continuous everywhere

5. If the sides of a square are decreasing at the rate of 1·5 cm/s, the rate of decrease of its perimeter is :

(A) 1·5 cm/s (B) 6 cm/s

(C) 3 cm/s (D) 2·25 cm/s

6. $\int_{-a}^{a} f(x)\,dx = 0$, if :

(A) $f(-x) = f(x)$ (B) $f(-x) = -f(x)$

(C) $f(a - x) = f(x)$ (D) $f(a - x) = -f(x)$

7. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

8. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

9. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

10. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

(C) optimal solutions (D) infeasible solutions

11. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F \mid E)$ is :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

12. If A and B are two skew symmetric matrices, then (AB + BA) is :

(A) a skew symmetric matrix (B) a symmetric matrix

(C) a null matrix (D) an identity matrix

13. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

(A) 2 (B) -2 (C) ± 2 (D) ∓ 2

14. The derivative of 2^x w.r.t. 3^x is :

(A) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ (B) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$

(C) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ (D) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

15. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

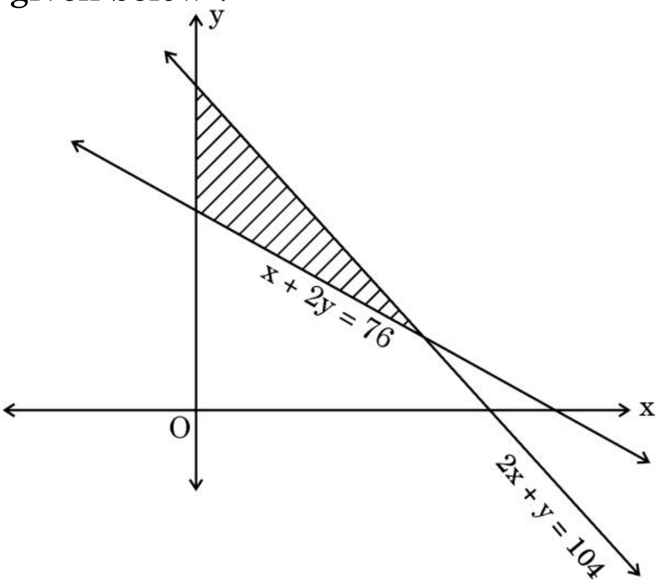
(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$

16. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

17. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \leq 76,\ 2x + y \geq 104,\ x, y \geq 0$

(B) $x + 2y \leq 76,\ 2x + y \leq 104,\ x, y \geq 0$

(C) $x + 2y \geq 76,\ 2x + y \leq 104,\ x, y \geq 0$

(D) $x + 2y \geq 76,\ 2x + y \geq 104,\ x, y \geq 0$

18. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

20. *Assertion (A) :* Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Evaluate :

$$\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$$

22. (a) If $x = e^{x/y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

23. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

24. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

25. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.
Prove that $|\vec{a} \times \vec{b}| \leq |\vec{a}| |\vec{b}|$.
State the condition under which equality holds, i.e., $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $x \cos (p + y) + \cos p \sin (p + y) = 0$, prove that $\cos p \frac{dy}{dx} = -\cos^2 (p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

27. (a) Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval [1, 2].

28. Find :

$$\int \frac{x^2+1}{(x^2+2)(x^2+4)}\,dx$$

29. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x}\,e^x\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\,dx$$

30. Solve the following linear programming problem graphically :

Maximise $z = 4x + 3y$,

subject to the constraints

$x + y \le 800$

$2x + y \le 1000$

$x \le 400$

$x, y \ge 0.$

31. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. A relation R on set A = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

33. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

34. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines x = – 2 and x = 2.

35. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

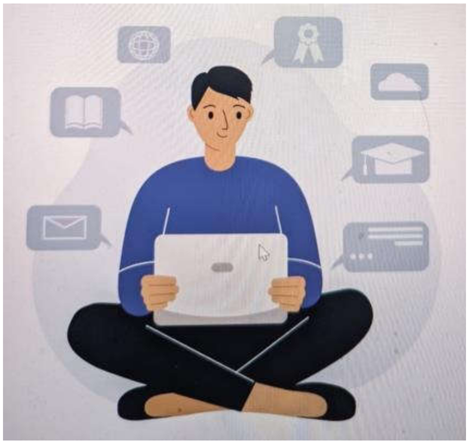
(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

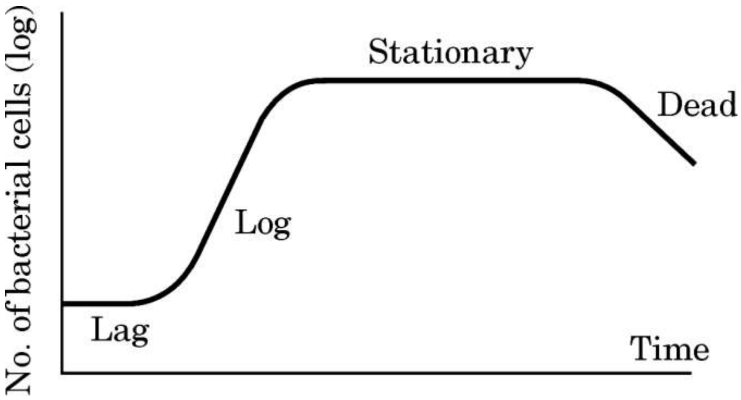
(iii) (a) Find the mean number of hours spent by the student. *2*

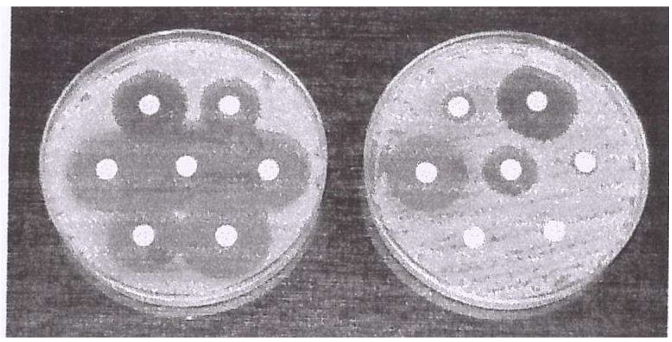
OR

(iii) (b) Find $P(1 < X < 6)$. *2*

Case Study – 2

37. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt}$ = kP, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2*

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*

Case Study – 3

38. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

2024 Annual

Series PQ3RS/3 **Set – 2**

प्रश्न-पत्र कोड
Q.P. Code **65/3/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*

Time allowed : 3 hours — *Maximum Marks : 80*

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General Instructions :

Read the following instructions very carefully and strictly follow them :

*(i) This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

*(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

*(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

*(vii) In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

*(ix) Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

(A) 2 (B) -2 (C) ± 2 (D) ∓ 2

2. The derivative of 5^x w.r.t. e^x is :

(A) $\left(\frac{5}{e}\right)^x \frac{1}{\log 5}$ (B) $\left(\frac{e}{5}\right)^x \frac{1}{\log 5}$

(C) $\left(\frac{5}{e}\right)^x \log 5$ (D) $\left(\frac{e}{5}\right)^x \log 5$

3. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$
~~~
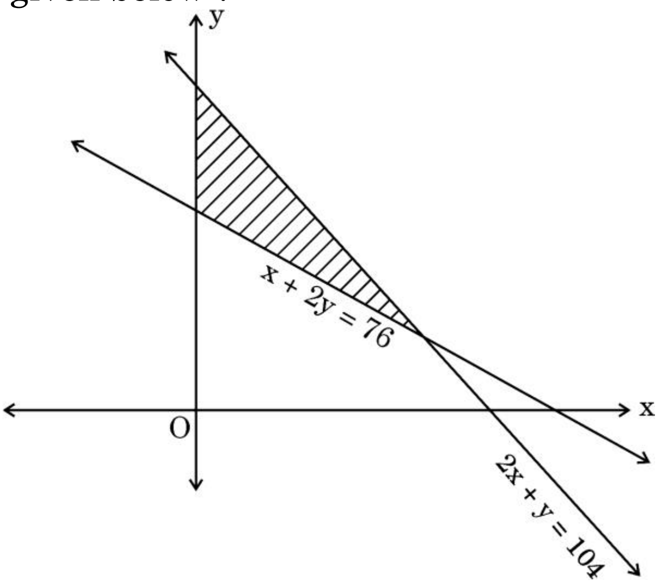

4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

5. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \le 76,\ 2x + y \ge 104,\ x, y \ge 0$

(B) $x + 2y \le 76,\ 2x + y \le 104,\ x, y \ge 0$

(C) $x + 2y \ge 76,\ 2x + y \le 104,\ x, y \ge 0$

(D) $x + 2y \ge 76,\ 2x + y \ge 104,\ x, y \ge 0$

6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

7. For any square matrix A, $(A - A')$ is always

(A) an identity matrix

(B) a null matrix

(C) a skew symmetric matrix

(D) a symmetric matrix

8. A function $f : R \rightarrow A$ defined as $f(x) = x^2 + 1$ is onto, if A is :

(A) $(-\infty, \infty)$ (B) $(1, \infty)$

(C) $[1, \infty)$ (D) $[-1, \infty)$

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

(A) 2a (B) 2b

(C) 2c (D) 0

10. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at $x = 1$ only (B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$ (D) continuous everywhere

11. The point of inflexion of a function f(x) is the point where

(A) $f'(x) = 0$ and $f'(x)$ changes its sign from positive to negative from left to right of that point.

(B) $f'(x) = 0$ and $f'(x)$ changes its sign from negative to positive from left to right of that point.

(C) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point.

(D) $f'(x) \neq 0$.

12. If g(x) is a continuous function satisfying $g(-x) = -g(x)$, then $\int_0^{2a} g(x)\,dx$ is equal to :

(A) 0 (B) $2\int_0^{a} g(x)\,dx$

(C) $\int_{-a}^{a} g(x)\,dx$ (D) $-\int_{-2a}^{0} g(x)\,dx$

13. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

15. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

(C) optimal solutions (D) infeasible solutions

17. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F | E)$ is :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

18. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$ (B) $a_{ij} = 1, \forall\, i, j$

(C) $a_{ij} = 0, \forall\, i, j$ (D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

20. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

22. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

23. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} - \vec{c} = \vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.

24. Find the value of $\left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\} + \tan^2\left\{\sec^{-1}(3)\right\}\right]$.

25. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \le x < 1 \\ 3-x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^{-1}}{(\log x)^2 - 5\log x + 4}\, dx$$

27. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x} e^x\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\, dx$$

28. Solve the following linear programming problem graphically :

Minimize $z = 600x + 400y$,

subject to the constraints

$$x + y \geq 8$$
$$x + 2y \leq 16$$
$$4x + y \leq 29$$
$$x, y \geq 0.$$

29. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

30. (a) If $x \cos (p + y) + \cos p \sin (p + y) = 0$, prove that $\cos p \dfrac{dy}{dx} = - \cos^2 (p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \dfrac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

31. (a) Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \dfrac{x}{2} + \dfrac{2}{x}$, on the interval [1, 2].

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

33. Find the area of the region bounded by the lines $x - 2y = 4$, $x = -1$, $x = 6$ and x-axis, using integration.

34. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

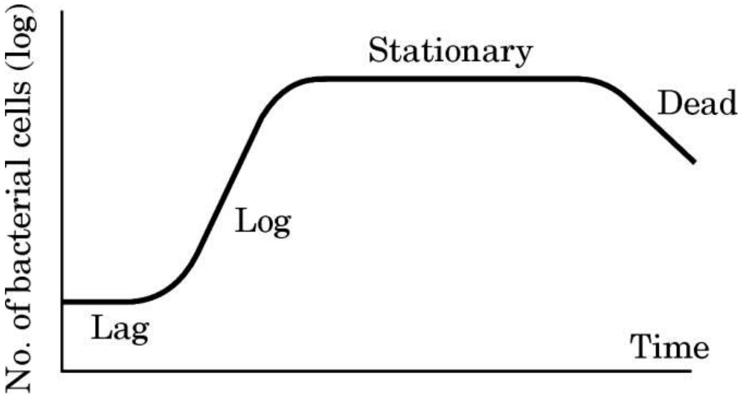
35. A relation R on set $A = \{x : -10 \le x \le 10, x \in Z\}$ is defined as $R = \{(x, y) : (x - y) \text{ is divisible by } 5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5].

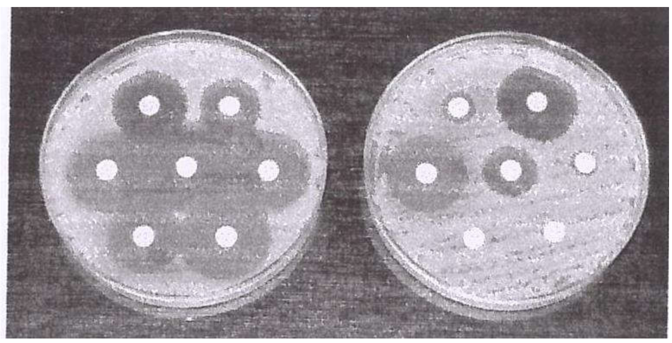
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt}$ = kP, where P is the population of bacteria at any time ‘t’.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of ‘t’. *2*

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*

Case Study – 2

37. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

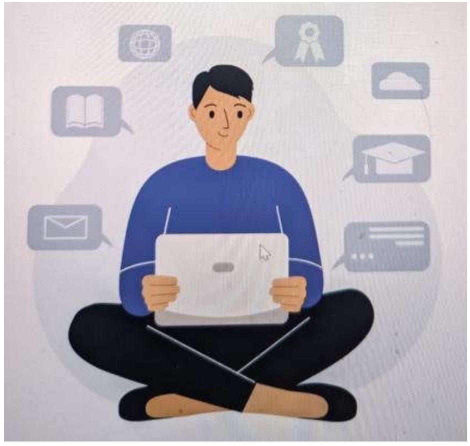
(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

Case Study – 3

38. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

(iii) (a) Find the mean number of hours spent by the student. *2*

OR

(iii) (b) Find $P(1 < X < 6)$. *2*

2024 Annual

Series PQ3RS/3

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/3/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

*(i) This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

*(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

*(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

*(vii) In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

*(ix) Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The value of $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ is :

(A) 0 (B) 2

(C) 7 (D) – 2

2. If $y = \sin^{-1} x$, then $\frac{d^2y}{dx^2}$ is :

(A) $\sec y$ (B) $\sec y \tan y$

(C) $\sec^2 y \tan y$ (D) $\tan^2 y \sec y$

3. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

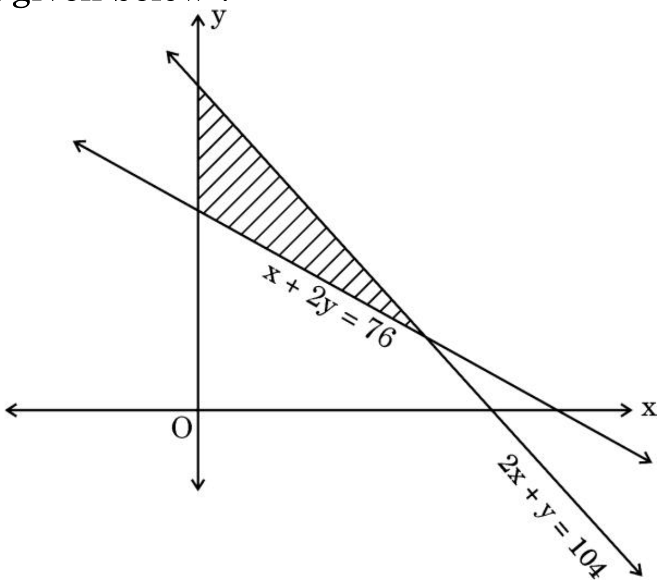
(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$

4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

5. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \leq 76,\ 2x + y \geq 104,\ x, y \geq 0$

(B) $x + 2y \leq 76,\ 2x + y \leq 104,\ x, y \geq 0$

(C) $x + 2y \geq 76,\ 2x + y \leq 104,\ x, y \geq 0$

(D) $x + 2y \geq 76,\ 2x + y \geq 104,\ x, y \geq 0$

6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

7. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$

(B) $a_{ij} = 1, \forall\, i, j$

(C) $a_{ij} = 0, \forall\, i, j$

(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

8. Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is :

(A) both one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

(A) 2a

(B) 2b

(C) 2c

(D) 0

10. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at $x = 1$ only

(B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$

(D) continuous everywhere

11. The rate of change of surface area of a sphere with respect to its radius 'r', when r = 4 cm, is :

(A) 64π cm^2/cm

(B) 48π cm^2/cm

(C) 32π cm^2/cm

(D) 16π cm^2/cm

12. $\int_{-a}^{a} f(x)\, dx = 0$, if :

(A) $f(-x) = f(x)$

(B) $f(-x) = -f(x)$

(C) $f(a - x) = f(x)$

(D) $f(a - x) = -f(x)$

13. $x \log x \dfrac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

15. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

(C) optimal solutions (D) infeasible solutions

17. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F|E)$ is :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

18. If A and B are two skew symmetric matrices, then (AB + BA) is :

(A) a skew symmetric matrix (B) a symmetric matrix

(C) a null matrix (D) an identity matrix

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any non-zero unit vector $\vec{a}$, $\vec{a} \cdot (-\vec{a}) = (-\vec{a}) \cdot \vec{a} = -1$.

Reason (R) : Angle between $\vec{a}$ and $(-\vec{a})$ is $\frac{\pi}{2}$.

20. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of cos θ.

22. Evaluate :

$$\cot^2\left\{\operatorname{cosec}^{-1} 3\right\} + \sin^2\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$$

23. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at $x = 1$.

24. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x - x^2}}$ and $F(1) = 0$, find $F(x)$.

25. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically :

Maximize $z = x + y$

subject to constraints

$$2x + 5y \leq 100$$

$$8x + 5y \leq 200$$

$$x \geq 0, y \geq 0.$$

27. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

28. (a) If x cos (p + y) + cos p sin (p + y) = 0, prove that

$\cos p \dfrac{dy}{dx} = -\cos^2 (p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \dfrac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

29. (a) Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \dfrac{x}{2} + \dfrac{2}{x}$, on the interval [1, 2].

30. Find :

$$\int \frac{\sqrt{x}}{(x+1)(x-1)} dx$$

31. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

33. Let A = R – {3} and B = R – {a}. Find the value of 'a' such that the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

34. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

35. Using integration, find the area of the region enclosed between the curve $y = \sqrt{4-x^2}$ and the lines x = –1, x = 1 and the x-axis.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

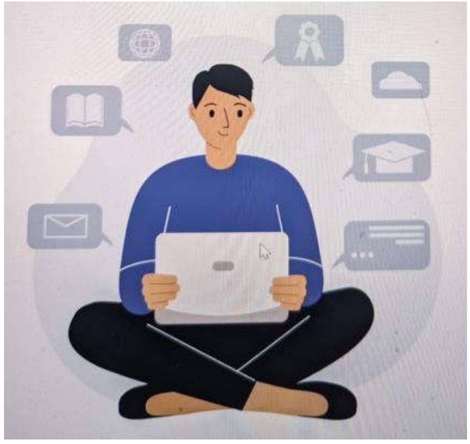
(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

Case Study – 2

37. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

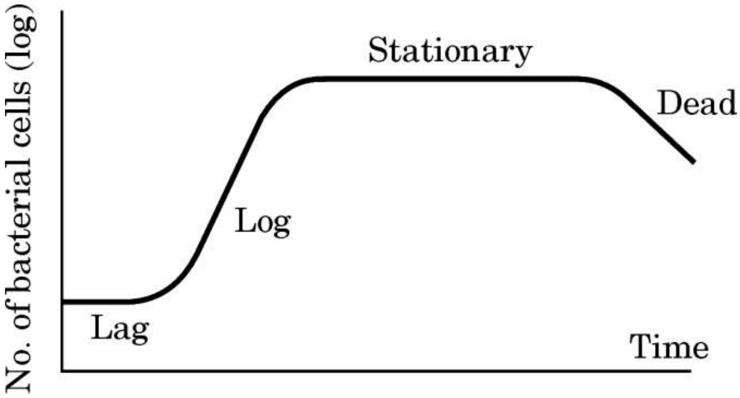
(iii) (a) Find the mean number of hours spent by the student. *2*

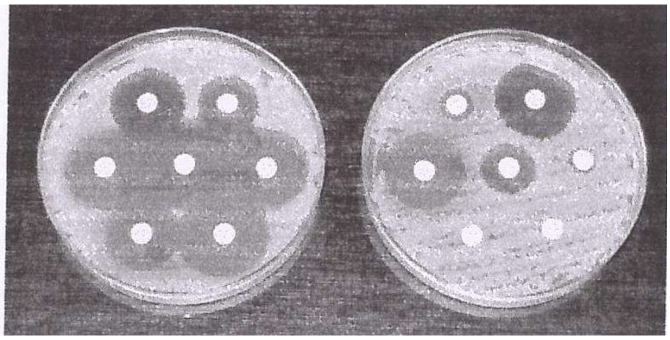
OR

(iii) (b) Find $P(1 < X < 6)$. *2*

Case Study – 3

38. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt}$ = kP, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2*

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k. *2*

2024 Annual

Series QSS4R/4

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/4/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
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- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
- *Please check that this question paper contains **23** printed pages.*
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गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question Paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question Paper is divided into **five** Sections – Section **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A** – Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Questions no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Questions no. **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **3** questions in Section **D** and **2** questions in Section **E**.*

(ix) *Use of calculators is **not** allowed.*

SECTION – A

This section consists of **20** multiple choice questions of **1** mark each. **$20 \times 1 = 20$**

1. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is :

(A) 0 (B) 5

(C) 10 (D) 25

2. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|\,A\,(\text{adj. }A)\,|$ is :

(A) 100 I

(B) 10 I

(C) 10

(D) 1000

5. Given that $[1 \quad x]\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is :

(A) –4

(B) –2

(C) 2

(D) 4

6. Derivative of e^{2x} with respect to e^x, is :

(A) e^x

(B) $2e^x$

(C) $2e^{2x}$

(D) $2e^{3x}$

7. For what value of k, the function given below is continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k \quad , & x = 0 \end{cases}$$

(A) 0 (B) $\frac{1}{4}$

(C) 1 (D) 4

8. The value of $\int_{0}^{3} \frac{dx}{\sqrt{9-x^2}}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{18}$

9. The general solution of the differential equation $x\,dy + y\,dx = 0$ is :

(A) $xy = c$ (B) $x + y = c$

(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

10. The integrating factor of the differential equation $(x + 2y^2) \frac{dy}{dx} = y$ $(y > 0)$ is :

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

11. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

12. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle

(B) an obtuse-angled triangle

(C) an isosceles triangle

(D) a right-angled triangle

13. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 \text{ is :}$$

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) 0

14. The vector equation of a line passing through the point (1, –1, 0) and parallel to Y-axis is :

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$ (B) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$

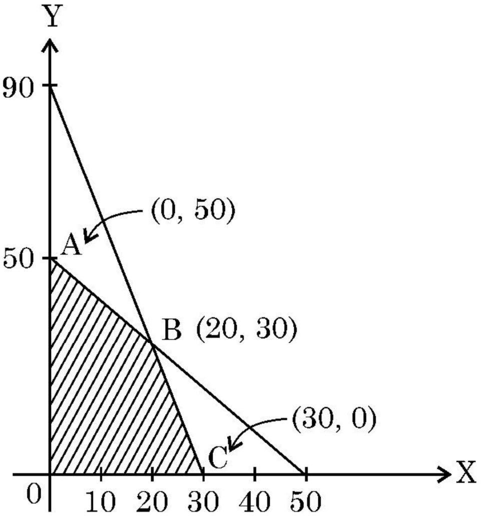
(C) $\vec{r} = \hat{i} - \hat{j} + \lambda\hat{k}$ (D) $\vec{r} = \lambda\hat{j}$

15. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

16. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is :



(A) 50 (B) 110

(C) 120 (D) 170

17. The probability distribution of a random variable X is :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{5}$ (D) 1

18. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

ASSERTION-REASON BASED QUESTIONS

Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

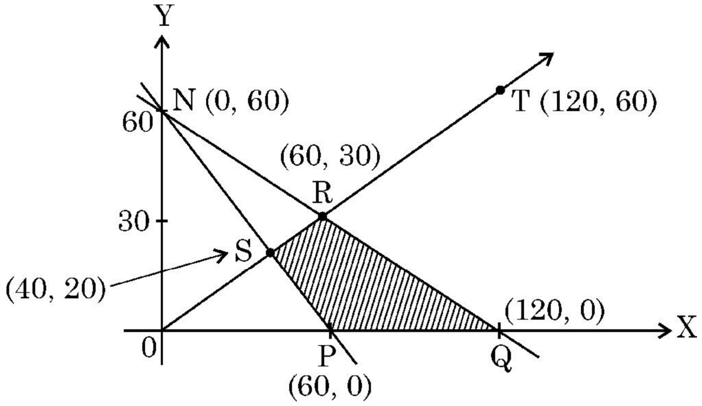
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

    Reason (R) : The number '2n' is composite for all natural numbers n.

20. **Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

SECTION – B

In this section there are **5** very short answer type questions of **2** marks each.

21. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

22. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

23. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

24. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

25. Find : $\int \frac{1}{x(x^2-1)}\, dx$.

SECTION – C

In this section there are **6** short answer type questions of **3** marks each.

26. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\frac{dy}{dx}$.

27. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\, dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

28. Find : $\int \frac{3x+5}{\sqrt{x^2+2x+4}}\, dx$

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$\left(xe^{\frac{y}{x}} + y\right) dx = x\, dy, \text{ given that } y = 1 \text{ when } x = 1.$$

30. Solve the following linear programming problem graphically :

Maximise $Z = 2x + 3y$

subject to the constraints :

$x + y \leq 6$

$x \geq 2$

$y \leq 3$

$x, y \geq 0$

31. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

SECTION – D

In the section there are **4** long answer type questions of **5** marks each.

32. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

33. (a) Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \dfrac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by $S = \{(a, b) : \text{where } a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive.

34. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations :

$2x + y - 3z = 13$

$3x + 2y + z = 4$

$x + 2y - z = 8$

35. (a) Find the distance between the line $\dfrac{x}{2} = \dfrac{2y-6}{4} = \dfrac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

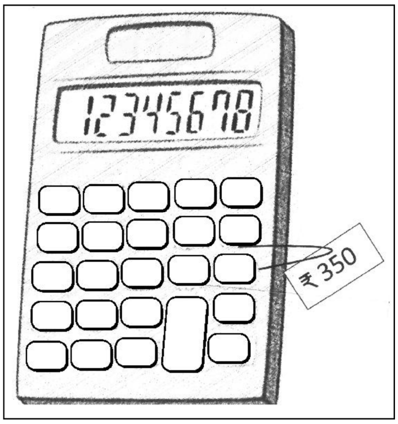
OR

(b) If the lines $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

SECTION – E

In this section, there are **3** case study based questions of **4** marks each.

36. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

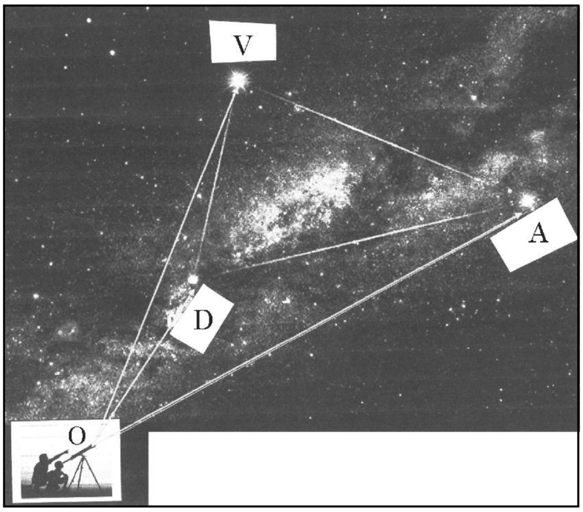


Based on the above information, answer the following questions :

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

37. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ? **1**

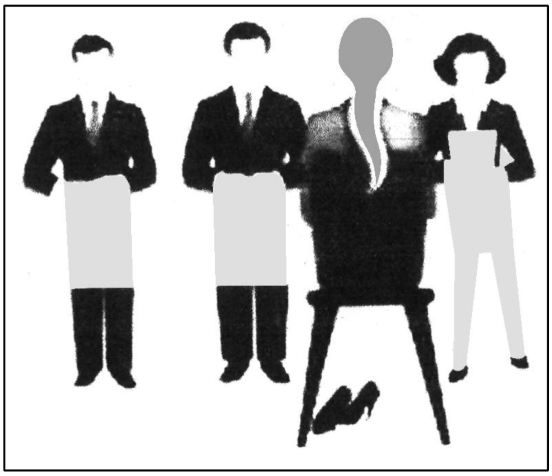
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle$VDA. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

38. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ? **1**

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

2024 Annual

Series QSS4R/4

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/4/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question Paper contains* ***38*** *questions.* ***All*** *questions are* **compulsory**.

(ii) *Question Paper is divided into* ***five*** *Sections – Section* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A*** *– Questions no.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and Questions no.* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B*** *– Questions no.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C*** *– Questions no.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D*** *– Questions no.* ***32*** *to* ***35*** *are Long Answer (LA) type questions, carrying* ***5*** *marks each.*

(vii) *In* ***Section E*** *– Questions no.* ***36*** *to* ***38*** *are case study based questions, carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section* ***B****,* ***3*** *questions in Section* ***C****,* ***3*** *questions in Section* ***D*** *and* ***2*** *questions in Section* ***E****.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION – A

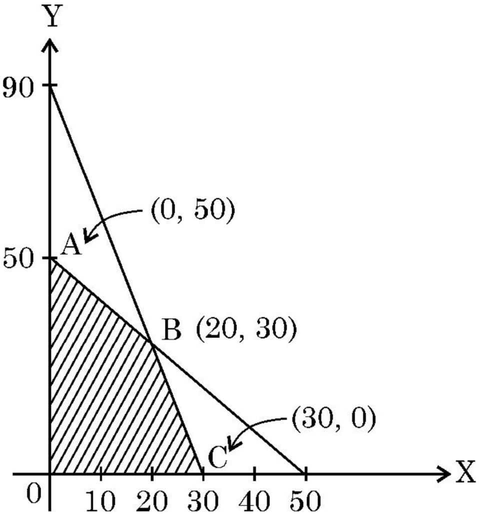
This section consists of **20** multiple choice questions of **1** mark each. **20 × 1 = 20**

1. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

2. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is :



(A) 50 (B) 110

(C) 120 (D) 170

3. The probability distribution of a random variable X is :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{5}$ (D) 1

4. If $A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ and c_{ij} is the cofactor of element a_{ij}, then the value of $a_{21} \cdot c_{11} + a_{22} \cdot c_{12} + a_{23} \cdot c_{13}$ is :

(A) –57 (B) 0

(C) 9 (D) 57

5. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then the value of k is :

(A) 3 (B) 5

(C) 7 (D) 9

6. If $e^{x^2 y} = c$, then $\dfrac{dy}{dx}$ is :

(A) $\dfrac{x e^{x^2 y}}{2y}$ (B) $\dfrac{-2y}{x}$

(C) $\dfrac{2y}{x}$ (D) $\dfrac{x}{2y}$

7. The value of constant c that makes the function f defined by

$$f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$$

continuous for all real numbers is :

(A) –2 (B) –1

(C) 0 (D) 2

8. The value of $\int_{-1}^{1} |x| \, dx$ is :

(A) –2 (B) –1

(C) 1 (D) 2

9. The number of arbitrary constants in the particular solution of the differential equation

$\log\left(\dfrac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ is/are

(A) 2 (B) 1

(C) 0 (D) 3

10. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + 2b + 3c + 4d is :

(A) 0 (B) 5

(C) 10 (D) 25

11. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \dots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

13. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y \ (y > 0)$ is :

(A) $\frac{1}{x}$

(B) x

(C) y

(D) $\frac{1}{y}$

14. A vector perpendicular to the line $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ is :

(A) $5\hat{i} + \hat{j} + 6k$

(B) $\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $2\hat{i} - 2\hat{j}$

(D) $9\hat{i} - 3\hat{j}$

15. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle

(B) an obtuse-angled triangle

(C) an isosceles triangle

(D) a right-angled triangle

16. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 \text{ is :}$$

(A) a^2

(B) $2a^2$

(C) $3a^2$

(D) 0

17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

18. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

ASSERTION-REASON BASED QUESTIONS

Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below :

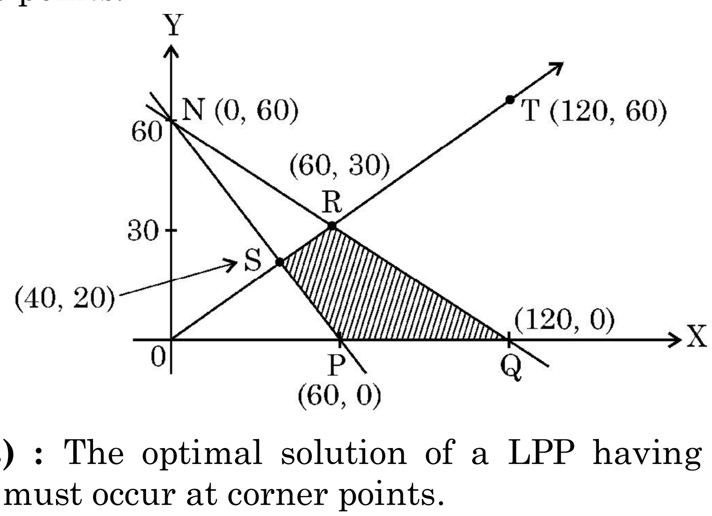
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

20. **Assertion (A) :** The relation R = $\{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

SECTION – B

In this section there are **5** very short answer type questions of **2** marks each.

21. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

22. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

23. Show that $f(x) = \frac{4\sin x}{2+\cos x} - x$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$.

24. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

25. Evaluate : $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1+x}{1-x}\right) dx$

SECTION – C

In this section there are **6** short answer type questions of **3** marks each.

26. Given that $x^y + y^x = a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$\left(xe^{\frac{y}{x}} + y\right) dx = x\, dy, \text{ given that } y = 1 \text{ when } x = 1.$$

28. Find : $\int \frac{2x+3}{x^2(x+3)}\, dx$

29. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

30. Solve the following L.P.P. graphically :

Maximise $Z = x + 3y$

subject to the constraints :

$x + 2y \leq 200$

$x + y \leq 150$

$y \leq 75$

$x, y \geq 0$

31. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right] dx$

SECTION – D

In the section there are **4** long answer type questions of **5** marks each.

32. (a) Let A = R – {5} and B = R – {1}. Consider the function f : A $\to$ B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where a – b + $\sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

33. (a) Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

OR

(b) If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

34. Use the product of matrices $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}$ to solve the following system of equations :

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

35. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

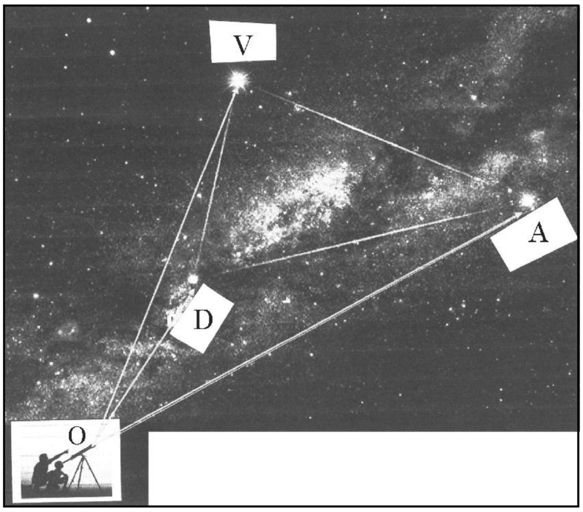
OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

SECTION – E

In this section, there are **3** case study based questions of **4** marks each.

36. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ? **1**

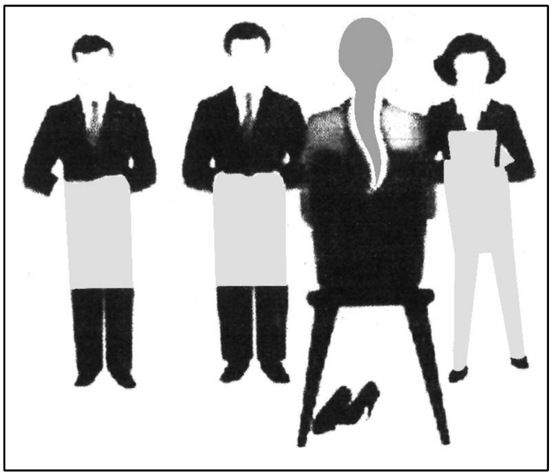
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle$VDA. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

37. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ? **1**

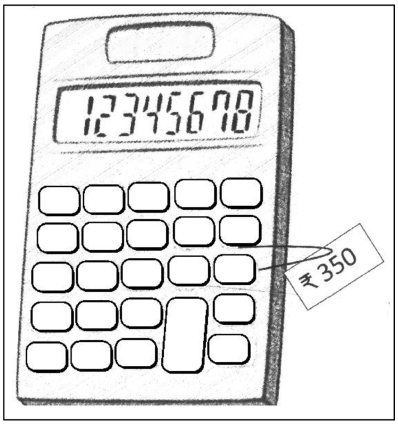
(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

38. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.



Based on the above information, answer the following questions :

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

2024 Annual

Series QSS4R/4

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/4/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***
*Time allowed : **3** hours* | *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

*(i) This Question Paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) Question Paper is divided into **five** Sections – Section **A, B, C, D** and **E**.*

*(iii) In **Section A** – Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

*(iv) In **Section B** – Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C** – Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D** – Questions no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

*(vii) In **Section E** – Questions no. **36** to **38** are case study based questions, carrying **4** marks each.*

*(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **3** questions in Section **D** and **2** questions in Section **E**.*

*(ix) Use of calculators is **not** allowed.*

SECTION – A

This section consists of **20** multiple choice questions of **1** mark each. **20 × 1 = 20**

1. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

2. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle (B) an obtuse-angled triangle

(C) an isosceles triangle (D) a right-angled triangle

3. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is :

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) 0

4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of k is :

(A) 1 (B) 2

(C) 5 (D) 7

5. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If AB = I, then the value of λ is :

(A) $\frac{-9}{4}$ (B) -2

(C) $\frac{-3}{2}$ (D) 0

6. Derivative of x^2 with respect to x^3, is :

(A) $\frac{2}{3x}$ (B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$ (D) $6x^5$

7. The function $f(x) = |x| + |x - 2|$ is

(A) continuous, but not differentiable at $x = 0$ and $x = 2$.

(B) differentiable but not continuous at $x = 0$ and $x = 2$.

(C) continuous but not differentiable at $x = 0$ only.

(D) neither continuous nor differentiable at $x = 0$ and $x = 2$.

8. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is :

(A) $\pi + \sqrt{3}$
(B) $3\sqrt{3} - \pi$
(C) $\sqrt{3} - \pi$
(D) $\pi - \sqrt{3}$

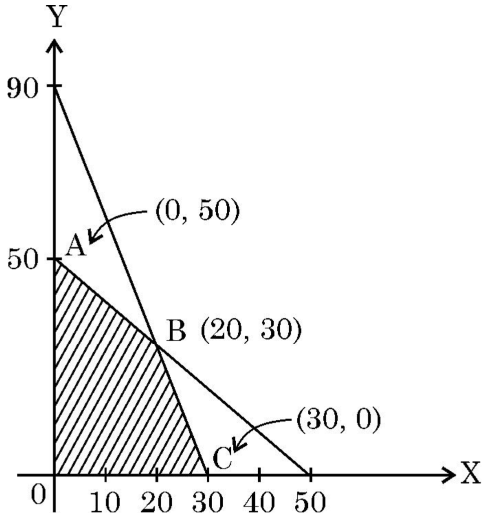
9. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0,\ x \neq 0$ is :

(A) $\frac{2}{x}$
(B) x^2
(C) $e^{\frac{2}{x}}$
(D) $e^{\log(2x)}$

10. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) 2
(D) 3

11. The maximum value of Z = $4x$ + y for a L.P.P. whose feasible region is given below is :



(A) 50
(B) 110
(C) 120
(D) 170

12. The probability distribution of a random variable X is :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{5}$ (D) 1

13. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

14. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is

(A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(C) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (D) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

15. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + 2b + 3c + 4d is :

(A) 0 (B) 5

(C) 10 (D) 25

16. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

17. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

18. The integrating factor of the differential equation $(x + 2y^2) \dfrac{dy}{dx} = y \ (y > 0)$ is :

(A) $\dfrac{1}{x}$

(B) x

(C) y

(D) $\dfrac{1}{y}$

ASSERTION-REASON BASED QUESTIONS

Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

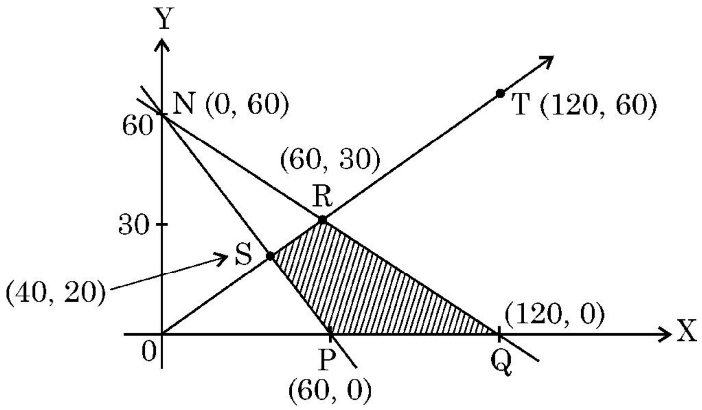
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

20. **Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of Z = x + 2y occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

SECTION – B

In this section there are **5** very short answer type questions of **2** marks each.

21. (a) If $y = \cos^3 (\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

22. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

23. Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

24. (a) Express $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$, where $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\dfrac{1}{2}\right) + \sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$.

25. Find : $\displaystyle\int \frac{2x}{(x^2+1)(x^2-4)}\, dx$.

SECTION – C

In this section there are **6** short answer type questions of **3** marks each.

26. Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

27. (a) Find the particular solution of the differential equation $\dfrac{dy}{dx} = y \cot 2x$, given that $y\left(\dfrac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$\left(xe^{\frac{y}{x}} + y\right) dx = x\, dy, \text{ given that } y = 1 \text{ when } x = 1.$$

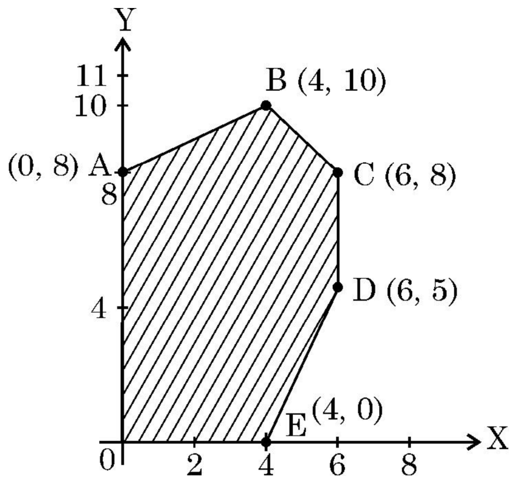
28. Find : $\displaystyle\int \sec^3\theta\, d\theta$

29. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

30. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure :



(i) If Z = $3x - 4y$ be the objective function, then find the maximum value of Z.

(ii) If Z = $px + qy$ where p, q > 0 be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

31. (a) Evaluate : $\displaystyle\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1 + \cos 2x + \sin 2x}$

OR

(b) Find : $\displaystyle\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

SECTION – D

In the section there are **4** long answer type questions of **5** marks each.

32. (a) Let A = R – {5} and B = R – {1}. Consider the function f : A $\to$ B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where $a - b + \sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

33. (a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

OR

(b) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

34. Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Hence, solve the following system of equations :

$x + 2y + z = 5$

$2x + 3y = 1$

$x - y + z = 8$

35. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

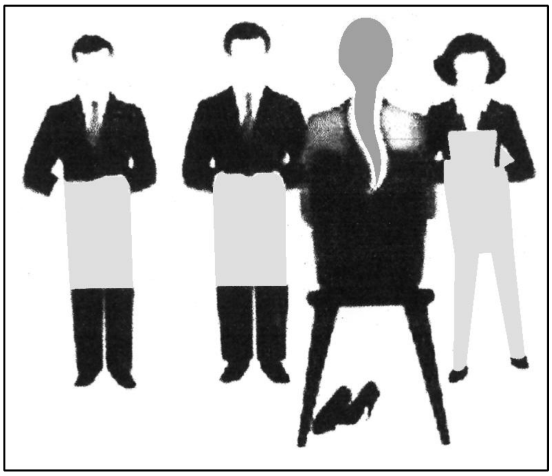
OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

SECTION – E

In this section, there are **3** case study based question of **4** marks each.

36. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ? **1**

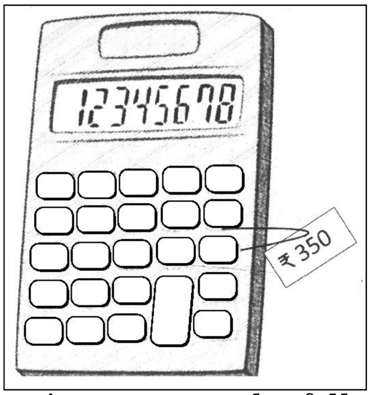
(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

37. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.

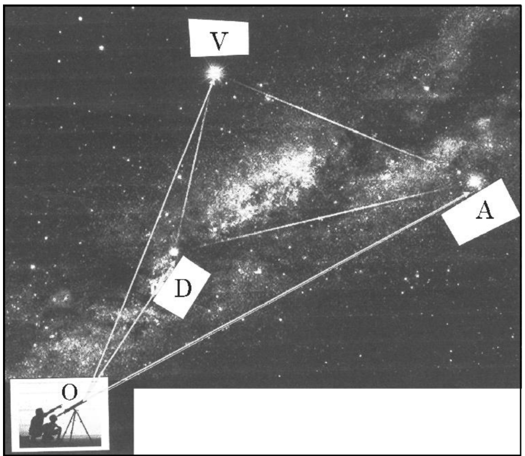


Based on the above information, answer the following questions :

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

38. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ? **1**

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle VDA$. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

2024 Annual

Series Q5QPS/5

Set – 1

प्रश्न-पत्र कोड
Q.P. Code **65/5/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B,** C, **D** and **E**.*

(iii) *In **Section A** – Questions Number **1** to **18** are Multiple Choice Questions (MCQs) type and Questions Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Questions Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Questions Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Questions Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Questions Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION – A

This section has **20** multiple choice questions of **1** mark each.

1. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

   (A) injective but not surjective. (B) surjective but not injective.

   (C) both injective and surjective. (D) neither injective nor surjective.

2. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is :

   (A) 0 (B) 1

   (C) –10 (D) 10

3. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :

   (A) $\sqrt{2}$ (B) $-\sqrt{2}$

   (C) 8 (D) $2\sqrt{2}$

4. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

   (A) –4 (B) 1

   (C) 3 (D) 4

5. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :

   (A) –1 (B) 0

   (C) 1 (D) 2

6. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

(A) -1 (B) 1

(C) $-e$ (D) $-\frac{1}{e}$

8. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x\, e^{\sin^2 x}$ (B) $\cos x\, e^{\sin^2 x}$

(C) $-2 \cos x\, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x\, e^{\sin^2 x}$

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2 (B) 1

(C) 0 (D) -2

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) -60 units/sec (B) 60 units/sec

(C) -70 units/sec (D) -140 units/sec

11. $\int \frac{1}{x(\log x)^2}\, dx$ is equal to :

(A) $2 \log (\log x) + c$ (B) $-\frac{1}{\log x} + c$

(C) $\frac{(\log x)^3}{3} + c$ (D) $\frac{3}{(\log x)^3} + c$

12. The value of $\int_{-1}^{1} x\,|x|\,dx$ is :

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $-\frac{1}{6}$ (D) 0

13. Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is :

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$

(C) 3 (D) $\frac{4}{3}$

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(A) 4 (B) 3

(C) 2 (D) not defined

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$

(C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

17. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(A) $A \subset B$, but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A)** **:** The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R) **:** Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION – B

This section has **5** Very Short Answer questions of **2** marks each.

21. Find value of k if

$$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}.$$

22. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\dfrac{1}{x}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

23. The area of the circle is increasing at a uniform rate of 2 cm^2/sec. How fast is the circumference of the circle increasing when the radius r = 5 cm ?

24. (a) Find : $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find : $\int \frac{1}{5 + 4x - x^2} \, dx$

25. Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

SECTION – C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

27. If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

28. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} \, dx$

OR

(b) Find : $\int \frac{2x+1}{(x+1)^2 (x-1)} \, dx$

29. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2} ; y(0) = 5.$$

OR

(b) Solve the following differential equation :

$$x^2 \, dy + y(x + y) \, dx = 0$$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer.

31. The random variable X has the following probability distribution where a and b are some constants :

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean E(X) = 3, then find values of a and b and hence determine $P(X \geq 3)$.

SECTION – D

There are **4** long answer questions in this section. Each question is of **5** marks.

32. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

34. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

35. Solve the following L.P.P. graphically :

Maximise $Z = 60x + 40y$

Subject to $x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x, y \geq 0$

SECTION – E

In this section there are **3** case study questions of **4** marks each.

36. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

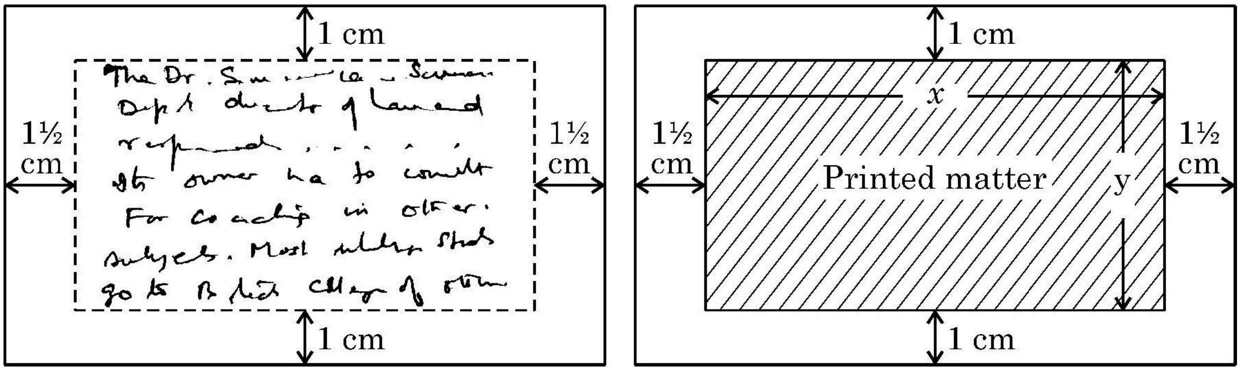
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.
Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

2024 Annual

Series Q5QPS/5

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/5/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
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गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* — *अधिकतम अंक : **80***

*Time allowed : **3** hours* — *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* **compulsory**.

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B****, C,* ***D*** *and* ***E****.*

(iii) *In* **Section** ***A*** *– Questions Number* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) type and Questions Number* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* **Section** ***B*** *– Questions Number* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* **Section** ***C*** *– Questions Number* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* **Section** ***D*** *– Questions Number* ***32*** *to* ***35*** *are Long Answer (LA) type questions, carrying* ***5*** *marks each.*

(vii) *In* **Section** ***E*** *– Questions Number* ***36*** *to* ***38*** *are case study based questions, carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section –* ***B****,* ***3*** *questions in Section –* ***C****, 2 questions in Section –* ***D*** *and* ***2*** *questions in Section –* ***E****.*

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A

This section has **20** multiple choice questions of **1** mark each.

1. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x \, e^{\sin^2 x}$ (B) $\cos x \, e^{\sin^2 x}$

(C) $-2 \cos x \, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x \, e^{\sin^2 x}$

2. If A is a square matrix of order 2 and $|A| = -2$, then value of $|5A'|$ is :

(A) -50 (B) -10

(C) 10 (D) 50

3. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2 (B) 1

(C) 0 (D) -2

4. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) -60 units/sec (B) 60 units/sec

(C) -70 units/sec (D) -140 units/sec

5. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3×2, then order of matrix P is :

(A) 2×2 (B) 3×3

(C) 2×3 (D) 3×2

6. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective. (B) surjective but not injective.

(C) both injective and surjective. (D) neither injective nor surjective.

7. If $\sin(xy) = 1$, then $\dfrac{dy}{dx}$ is equal to :

(A) $\dfrac{x}{y}$ (B) $-\dfrac{x}{y}$

(C) $\dfrac{y}{x}$ (D) $-\dfrac{y}{x}$

8. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(A) –4 (B) 1

(C) 3 (D) 4

9. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

10. If A is a square matrix of order 3 such that the value of $|\text{adj}\cdot A| = 8$, then the value of $|A^T|$ is :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) 8 (D) $2\sqrt{2}$

11. The value of $\displaystyle\int_{\pi/4}^{\pi/2} \cot\theta \,\text{cosec}^2\theta \, d\theta$ is :

(A) $\dfrac{1}{2}$ (B) $-\dfrac{1}{2}$

(C) 0 (D) $-\dfrac{\pi}{8}$

12. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to :

(A) $\frac{1}{6}\sin^{-1}\left(\frac{2x}{3}\right)+c$

(B) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

(C) $\sin^{-1}\left(\frac{2x}{3}\right)+c$

(D) $\frac{3}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$

13. The area of the region bounded by the curve $y^2 = 4x$ and $x = 1$ is :

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{64}{3}$

(D) $\frac{32}{3}$

14. The general solution of the differential equation

$\frac{dy}{dx} = e^{x+y}$ is :

(A) $e^x + e^{-y} = c$

(B) $e^{-x} + e^{-y} = c$

(C) $e^{x+y} = c$

(D) $2e^{x+y} = c$

15. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

16. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

17. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(A) $A \subset B$, but $A \neq B$

(B) $A = B$

(C) $A \cap B = \phi$

(D) $P(A) = P(B)$

18. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\dfrac{\vec{p} + 3\vec{q}}{4}$

(B) $\dfrac{\vec{p} + 3\vec{q}}{8}$

(C) $\dfrac{5\vec{p} + 3\vec{q}}{4}$

(D) $\dfrac{5\vec{p} + 3\vec{q}}{8}$

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

20. **Assertion (A)** : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

SECTION – B

This section has **5** Very Short Answer questions of **2** marks each.

21. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ and

$b = \tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

then find the value of a + b.

22. (a) Find : $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find : $\int \frac{1}{5 + 4x - x^2} \, dx$

23. Sand is pouring from a pipe at the rate of 15 cm^3/minute. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm ?

24. Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

25. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

SECTION – C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2} ; y(0) = 5.$$

OR

(b) Solve the following differential equation :

$$x^2 \, dy + y(x + y) \, dx = 0$$

27. Find the values of a and b so that the following function is differentiable for all values of x :

$$f(x) = \begin{cases} ax + b & , \quad x > -1 \\ bx^2 - 3 & , \quad x \leq -1 \end{cases}$$

28. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

29. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find : $\int \frac{2x + 1}{(x + 1)^2 (x - 1)} dx$

30. Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 3$.

31. Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

SECTION – D

There are **4** long answer questions in this section. Each question is of **5** marks.

32. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

33. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$x + 2y - 3z = 1$

$2x - 3z = 2$

$x + 2y = 3$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :

$x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

34. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

35. Solve the following Linear Programming problem graphically :

Maximise $Z = 300x + 600y$

Subject to $\quad x + 2y \leq 12$

$2x + y \leq 12$

$x + \frac{5}{4}y \geq 5$

$x \geq 0, y \geq 0.$

SECTION – E

In this section there are **3** case study questions of **4** marks each.

36. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.

Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

37. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

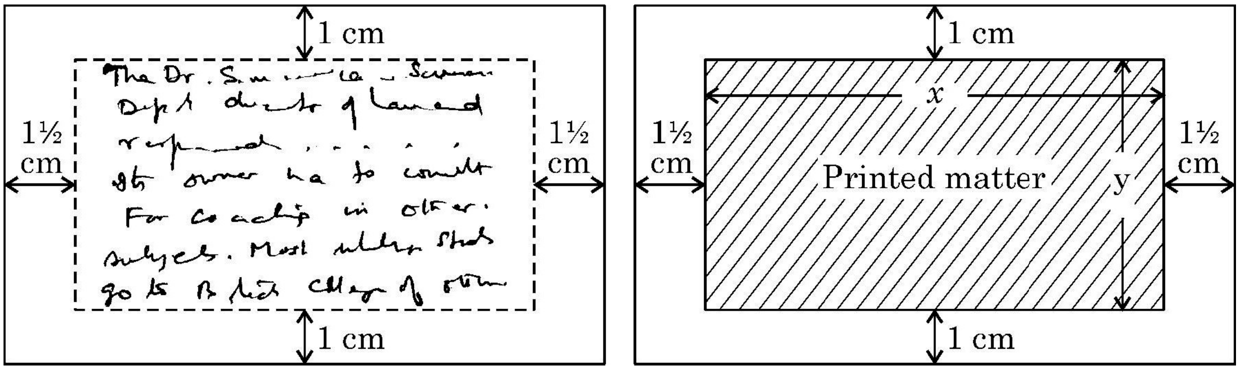
(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

38. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

Series Q5QPS/5

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/5/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* **compulsory**.

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B****, C,* ***D*** *and* ***E****.*

(iii) *In* **Section A** *– Questions Number* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) type and Questions Number* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* **Section B** *– Questions Number* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* **Section C** *– Questions Number* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* **Section D** *– Questions Number* ***32*** *to* ***35*** *are Long Answer (LA) type questions, carrying* ***5*** *marks each.*

(vii) *In* **Section E** *– Questions Number* ***36*** *to* ***38*** *are case study based questions, carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section –* ***B****,* ***3*** *questions in Section –* ***C****, 2 questions in Section –* ***D*** *and* ***2*** *questions in Section –* ***E****.*

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A

There are **20** multiple choice questions of **1** mark each in this section.

1. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

   (A) $\dfrac{\vec{p}+3\vec{q}}{4}$ (B) $\dfrac{\vec{p}+3\vec{q}}{8}$

   (C) $\dfrac{5\vec{p}+3\vec{q}}{4}$ (D) $\dfrac{5\vec{p}+3\vec{q}}{8}$

2. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is :

   (A) 0 (B) 10

   (C) $\mathbb{R} - \{10\}$ (D) $\mathbb{R} - \{-10\}$

3. The angle which the line $\dfrac{x}{1} = \dfrac{y}{-1} = \dfrac{z}{0}$ makes with the positive direction of Y-axis is :

   (A) $\dfrac{5\pi}{6}$ (B) $\dfrac{3\pi}{4}$

   (C) $\dfrac{5\pi}{4}$ (D) $\dfrac{7\pi}{4}$

4. The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

   $\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

   (A) $\dfrac{x-1}{2} = \dfrac{y+3}{0} = \dfrac{z-2}{-1}$ (B) $\dfrac{x+1}{1} = \dfrac{y-3}{1} = \dfrac{z+2}{2}$

   (C) $\dfrac{x+1}{2} = \dfrac{y-3}{0} = \dfrac{z+2}{-1}$ (D) $\dfrac{x-1}{1} = \dfrac{y+3}{1} = \dfrac{z-2}{2}$

5. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$, then value of x for which $A^2 = B$ is :

(A) -2 (B) 2

(C) 2 or -2 (D) 4

6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :

(A) -60 units/sec (B) 60 units/sec

(C) -70 units/sec (D) -140 units/sec

7. Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is :

(A) $\mathbb{R}$ (B) 1

(C) 0 (D) -1

8. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(A) $A \subset B$, but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

9. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective. (B) surjective but not injective.

(C) both injective and surjective. (D) neither injective nor surjective.

10. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) 8 (D) $2\sqrt{2}$

11. If $\int_{-2}^{3} x^2\,dx = k\int_{0}^{2} x^2\,dx + \int_{2}^{3} x^2\,dx$, then the value of k is :

(A) 2 (B) 1

(C) 0 (D) $\frac{1}{2}$

12. The value of $\int_{1}^{e} \log x\,dx$ is :

(A) 0 (B) 1

(C) e (D) e log e

13. The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines y = 0 and y = 3 is :

(A) $2\sqrt{3}$ (B) 27

(C) 9 (D) 3

14. The order of the following differential equation

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^5 = 4\log\left(\frac{d^4y}{dx^4}\right) \text{ is :}$$

(A) not defined (B) 3

(C) 4 (D) 5

15. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(A) –4 (B) 1

(C) 3 (D) 4

16. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by a_{ij} = maximum (i, j) – minimum (i, j) :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

17. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

(A) $\sin x \, e^{\sin^2 x}$ (B) $\cos x \, e^{\sin^2 x}$

(C) $-2 \cos x \, e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x \, e^{\sin^2 x}$

18. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :

(A) 2 (B) 1

(C) 0 (D) –2

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

20. **Assertion (A) :** The vectors

$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$

$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$

$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

SECTION – B

This section has **5** Very Short Answer questions of **2** marks each.

21. Simplify : $\cos^{-1}x + \cos^{-1}\left[\dfrac{x}{2} + \dfrac{\sqrt{3-3x^2}}{2}\right]$; $\dfrac{1}{2} \le x \le 1$

22. (a) Find : $\int \cos^3 x \, e^{\log \sin x} \, dx$

OR

(b) Find : $\int \dfrac{1}{5 + 4x - x^2} \, dx$

23. The surface area of a cube increases at the rate of 72 cm^2/sec. Find the rate of change of its volume, when the edge of the cube measures 3 cm.

24. Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

25. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x\sin\left(\dfrac{1}{x}\right) & , \; x \neq 0 \\ 0 & , \; x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

SECTION – C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Evaluate : $\displaystyle\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find : $\displaystyle\int \frac{2x+1}{(x+1)^2 (x-1)} dx$

27. If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 \dfrac{d^2y}{dx^2} + 2x(x^2 + 1)\dfrac{dy}{dx} = 2$.

28. (a) Find the particular solution of the differential equation

$\dfrac{dy}{dx} - 2xy = 3x^2 e^{x^2}$; $y(0) = 5$.

OR

(b) Solve the following differential equation :

$x^2 \, dy + y(x + y) \, dx = 0$

29. (a) Find $\dfrac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

30. Find the projection of vector $(\vec{b} + \vec{c})$ on vector $\vec{a}$, where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$.

31. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

SECTION – D

There are **4** long answer questions in this section. Each question is of **5** marks.

32. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

33. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

34. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$x + 2y - 3z = 1$

$2x - 3z = 2$

$x + 2y = 3$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

35. Solve the following L.P.P. graphically :

Minimise $Z = 6x + 3y$

Subject to constraints

$$4x + y \geq 80;$$

$$x + 5y \geq 115;$$

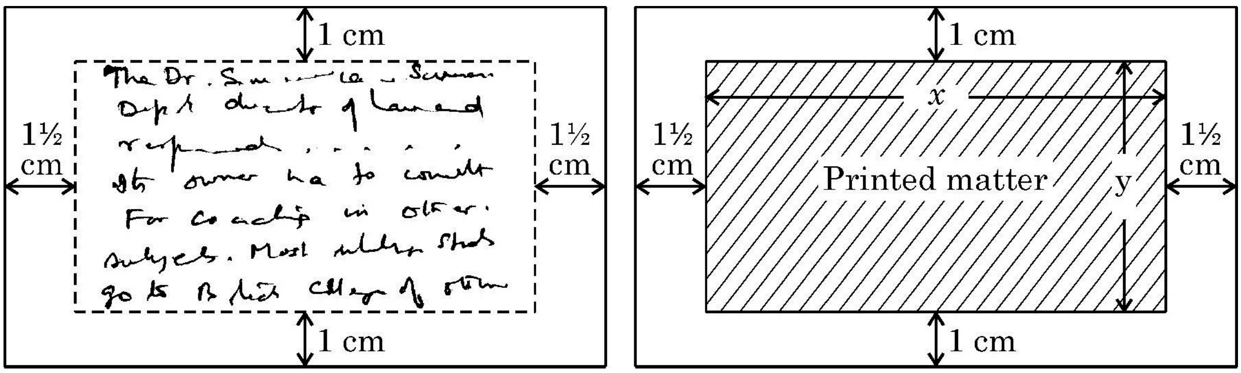
$$3x + 2y \leq 150$$

$$x, y \geq 0$$

SECTION – E

In this section there are **3** case study questions of **4** marks each.

36. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1½ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x.

(ii) Obtain the dimensions of the card of minimum area.

37. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.

Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

38. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

2023 Compt.

Series EF1GH/C **SET~1**

प्रश्न-पत्र कोड
Q.P. Code **65/C/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

2. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

3. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

5. A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

6. The vector equation of a line which passes through the point (2, – 4, 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

8. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

9. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) – 1 (d) 0

10. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1 (b) – 2 (c) 2 (d) – 1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

12. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

13. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

14. If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$

(b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$

(c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$

(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

15. It is given that $X\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$. Then matrix X is :

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$

16. If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is :

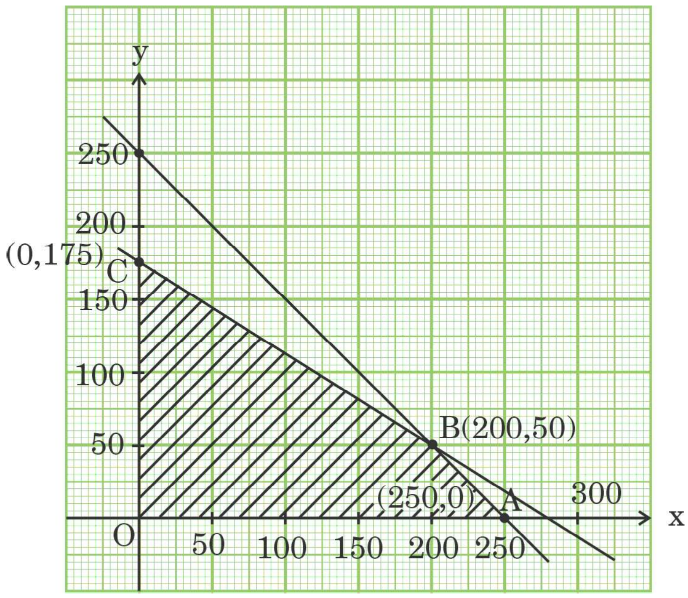
(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

17. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

18. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) 2a = b (b) 2a = 3b (c) a = b (d) a = 2b

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

20. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

22. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

23. Function f is defined as

$$f(x) = \begin{cases} 2x + 2, & \text{if} \quad x < 2 \\ k, & \text{if} \quad x = 2 \\ 3x, & \text{if} \quad x > 2 \end{cases}$$

Find the value of k for which the function f is continuous at x = 2.

24. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.

25. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x - 1| + |x - 2|) \, dx$$

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that y = 0 when x = 1.

28. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

29. Find :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

30. Solve the following Linear Programming Problem graphically :

Minimise z = 3x + 8y

subject to the constraints

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

31. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

33. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$.

34. Find the area of the triangle ABC bounded by the lines represented by the equations $5x - 2y - 10 = 0$, $x - y - 9 = 0$ and $3x - 4y - 6 = 0$, using integration method.

35. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

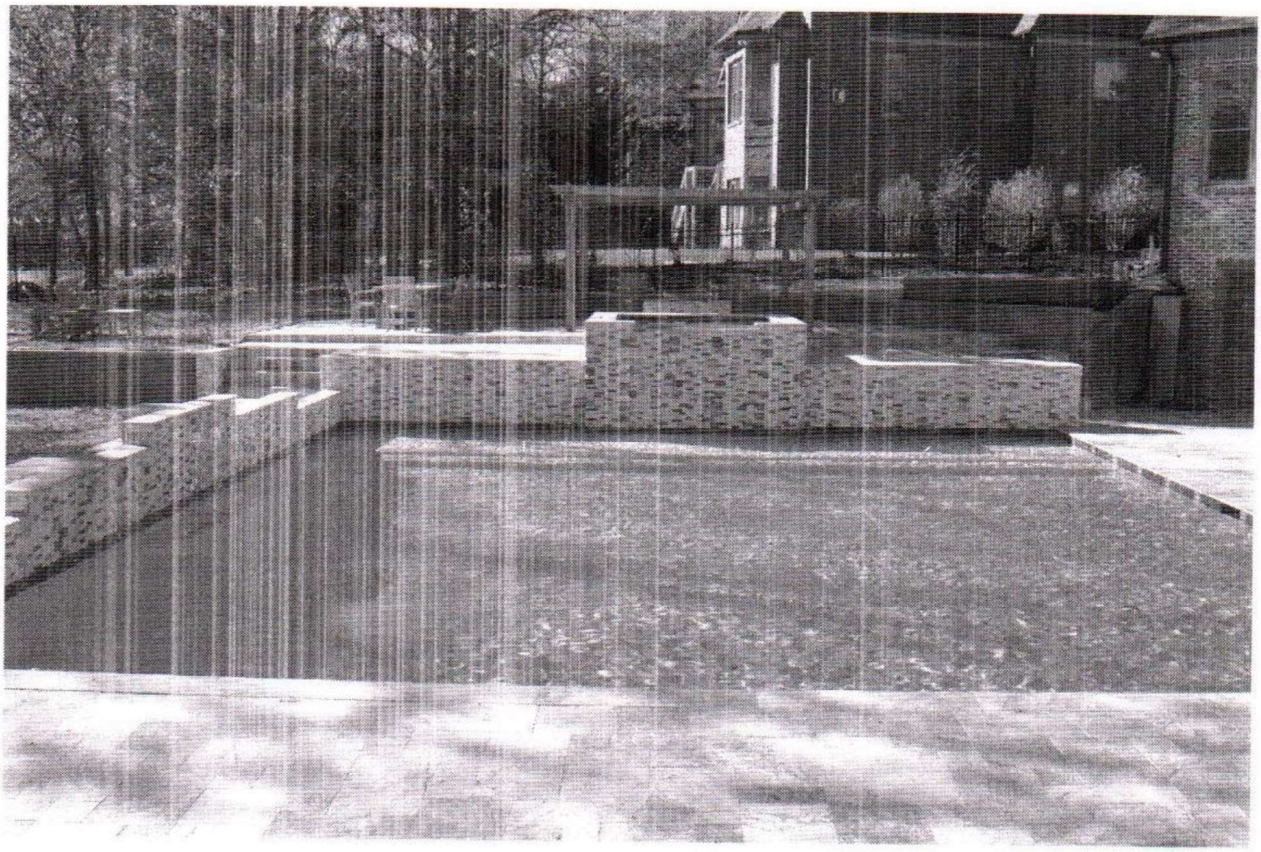
(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

Case Study – 2

37. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.



Suppose the side of the square plot is x metres and depth is h metres.

On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

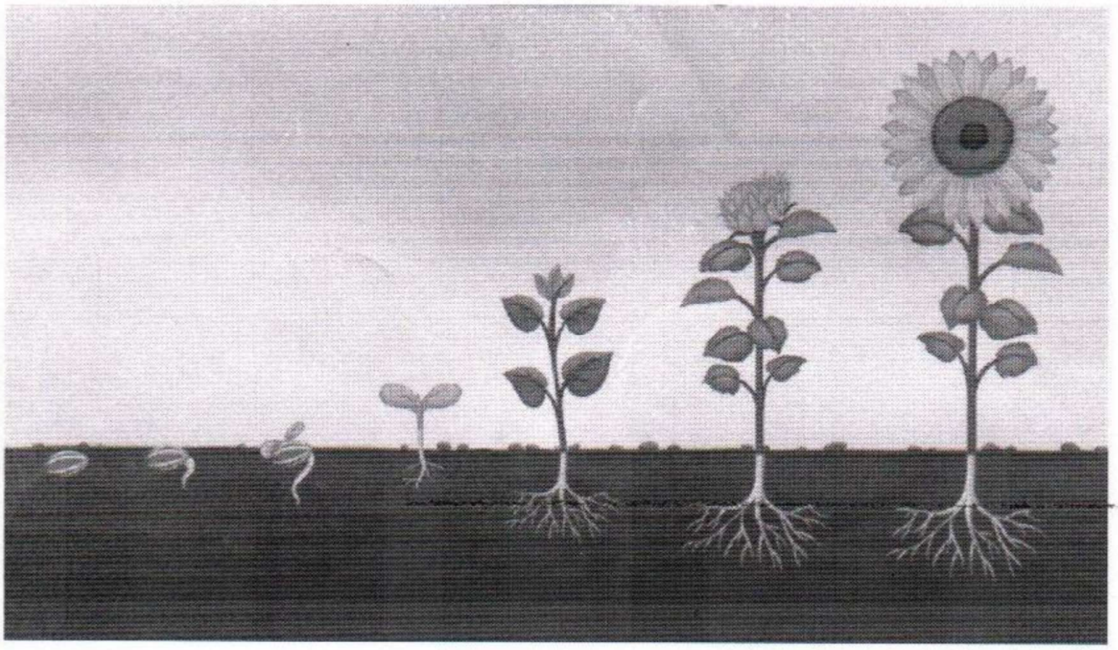
Case Study – 3

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ \ 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

2023 Compt.

Series EF1GH/C **SET~2**

प्रश्न-पत्र कोड
Q.P. Code **65/C/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*
Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

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(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Derivative of $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$ is :

(a) $-\dfrac{1}{4}$ (b) $\dfrac{1}{2}$

(c) 2 (d) $-\dfrac{1}{2}$

2. It is given that $X\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$. Then matrix X is :

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$

3. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix}4 & 3 & 2\\ 2 & -1 & 0\\ 1 & 2 & 3\end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

4. Solution of the differential equation $(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$ is :

(a) $\tan^{-1} y + \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(b) $\tan^{-1} y - \log |x| + \dfrac{(\log |x|)^2}{2} = C$

(c) $\tan^{-1} y - \log |x| - \dfrac{(\log |x|)^2}{2} = C$

(d) $\tan^{-1} y + \log |x| - \dfrac{(\log |x|)^2}{2} = C$

5. If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is :

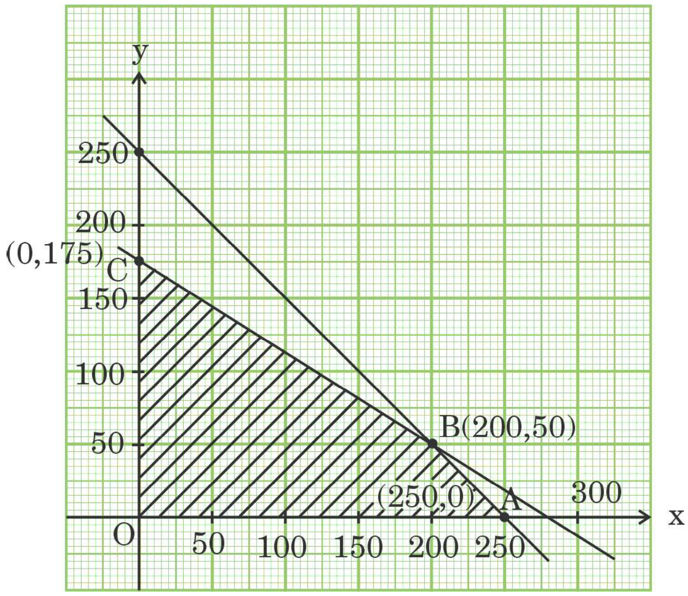
(a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$

6. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

(a) $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$ (b) $y^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

(c) $y^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ (d) $y^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$

7. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$

8. A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

9. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1 (b) -2 (c) 2 (d) -1

10. The vector equation of a line which passes through the point $(1, -2, 3)$ and is parallel to the vector $3\hat{i} - 2\hat{j} + 4\hat{k}$ is :

(a) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(b) $\vec{r} = (-3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

(c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(d) $\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

11. If $\begin{bmatrix} 3 & 2 \\ 1 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$, then x is :

(a) $\frac{16}{3}$ (b) -3

(c) -4 (d) 4

12. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

13. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

14. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

16. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

17. Let P and Q be two points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ respectively. The position vector of a point which divides the join of P and Q externally in the ratio 3 : 2 is :

(a) $4\vec{a} + 7\vec{b}$ (b) $\dfrac{8\vec{a} + 7\vec{b}}{5}$

(c) $4\vec{a} - 7\vec{b}$ (d) $\vec{a} + 4\vec{b}$

18. The difference of the order and the degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) -1 (d) 0

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\dfrac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

20. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Find the interval in which the function $x^3 - 12x^2 + 36x + 17$ is strictly increasing.

22. Find the points at which the function $f(x) = \dfrac{4 + x^2}{4x - x^3}$ is discontinuous.

23. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

24. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

25. For the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$$

27. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

28. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

29. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that y = 0 when x = 1.

30. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1| + |x-2|)\, dx$$

31. Solve the following Linear Programming Problem graphically:
Maximise z = 10x + 15y
subject to the constraints :

$$3x + 2y \leq 50$$
$$x + 4y \geq 20$$
$$x \geq 8, \; y \geq 0$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

33. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both a and b are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

34. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$x + y + z = 5000$$
$$6x + 7y + 8z = 35800$$
$$6x + 7y - 8z = 7000$$

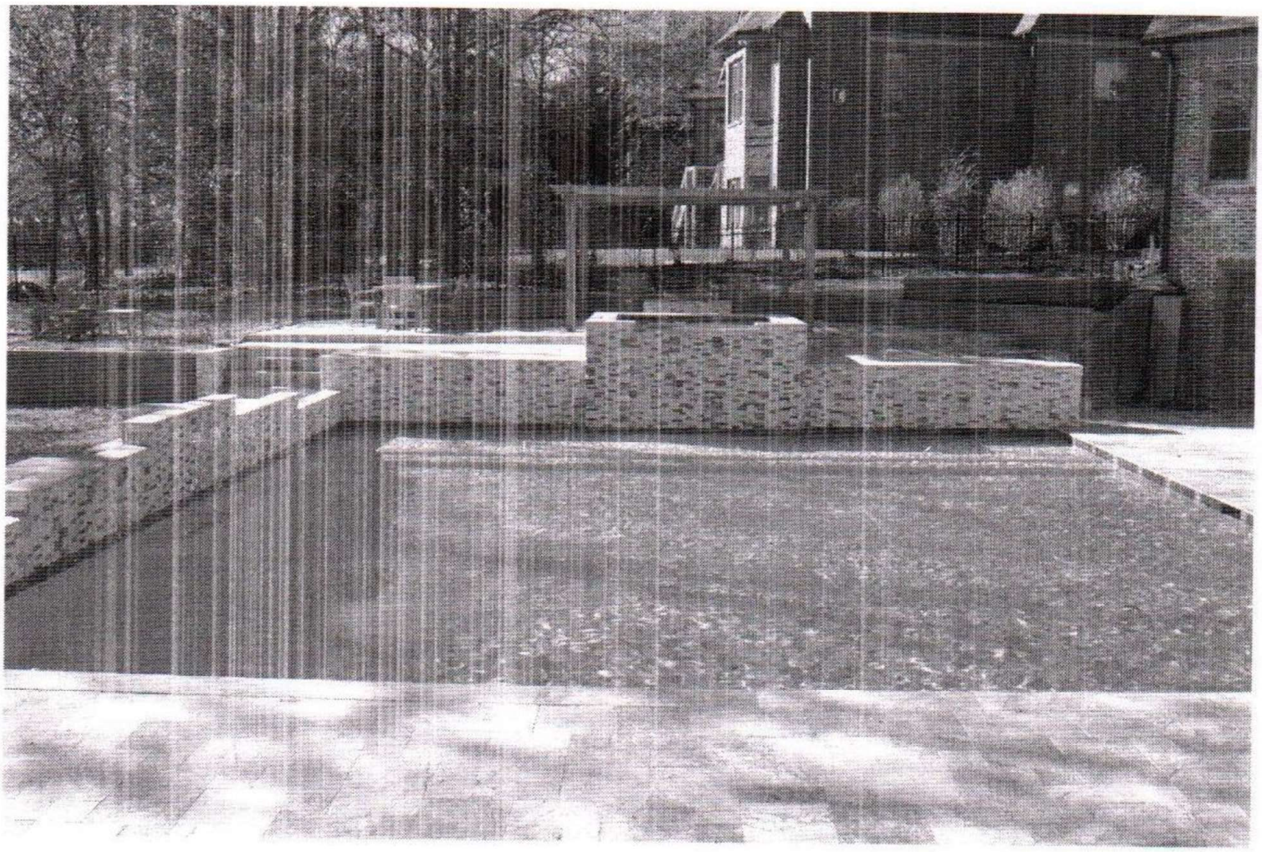
35. Using integration, find the area of the region bounded by the triangle ABC when its sides are given by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.



Suppose the side of the square plot is x metres and depth is h metres.
On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

Case Study – 2

37. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

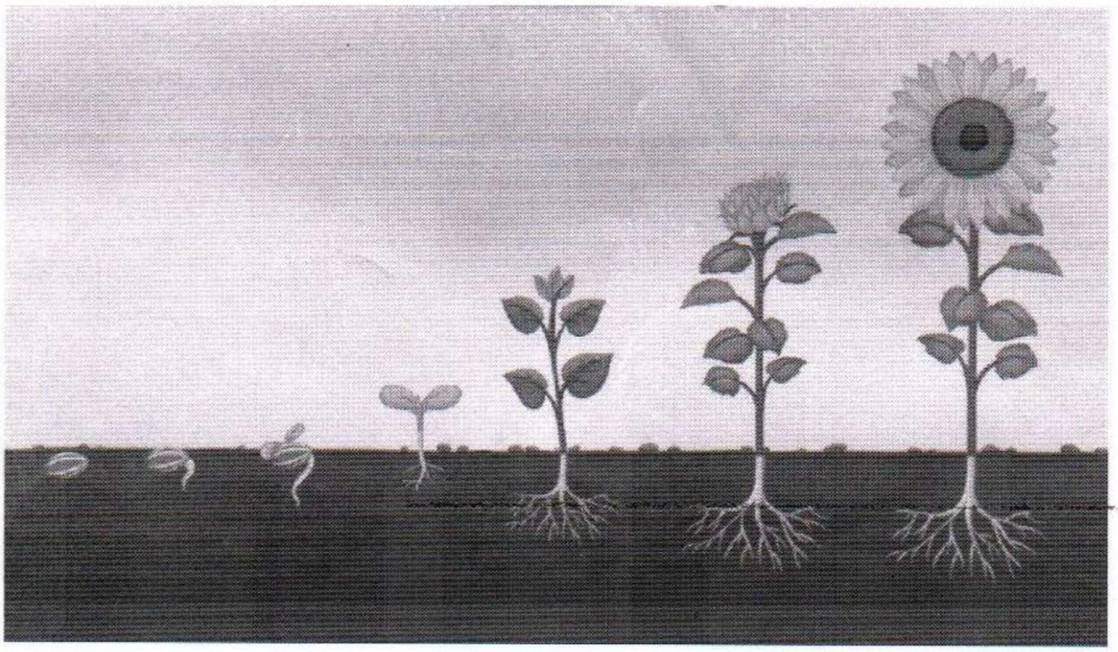
Case Study – 3

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \quad 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

2023 Compt.

Series EF1GH/C

SET~3

प्रश्न–पत्र कोड
Q.P. Code **65/C/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*

Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Integrating factor of the differential equation $x\frac{dy}{dx} - 2y = 4x^2$ is :

(a) x^2 (b) $-\frac{1}{x^2}$

(c) $\frac{1}{x^2}$ (d) $-x^2$

2. It is given that $X\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is :

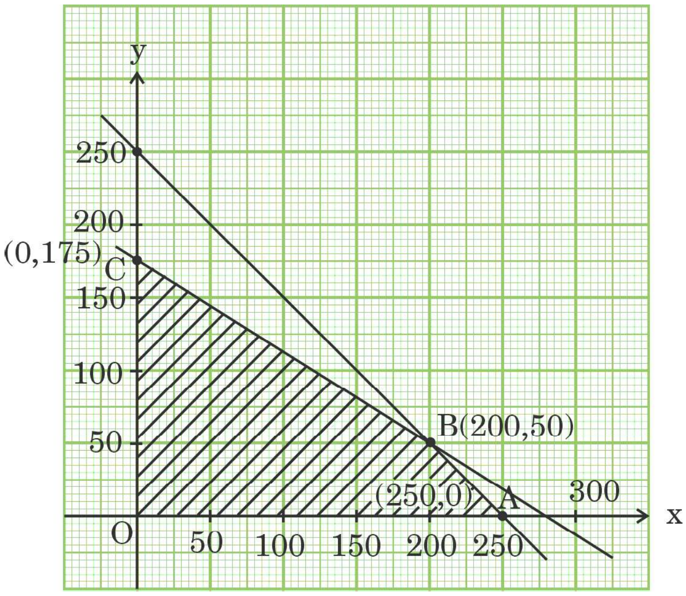
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

3. For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

4. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$

5. The angle between the lines $\frac{x+1}{1} = \frac{4-y}{-1} = \frac{z-5}{2}$ and $\frac{x+3}{-3} = \frac{y-2}{5} = \frac{z+5}{4}$ is :

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

6. A fair die is rolled. Events E and F are E = {1, 3, 5} and F = {2, 3} respectively. Value of $P(E|F)$ is :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{2}$

7. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

8. If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is :

(a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$

9. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then which one of the following is true ?

(a) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

10. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1 (b) -2 (c) 2 (d) -1

11. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) -1 (d) 0

12. $\int \frac{\cos 2x}{\sin^2 x \, . \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

13. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

14. In the matrix equation $\begin{bmatrix} x+y+z \\ x+z \\ y+2z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, the value of z is :

(a) 1 (b) 2

(c) -1 (d) -2

15. If $y = \log\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$, then $\frac{dy}{dx}$ is :

(a) sec x (b) cosec x

(c) tan x (d) sec x tan x

16. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

17. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

18. The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC $\neq$ BD.

20. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Find a vector of magnitude 6, which is perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

22. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

23. The function

$$f(x) = \begin{cases} 3ax + b, & \text{if} \quad x > 1 \\ 11, & \text{if} \quad x = 1 \\ 5ax - 2b, & \text{if} \quad x < 1 \end{cases}$$

is continuous at x = 1. Find the values of a and b.

24. Find the interval in which the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is decreasing.

25. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

27. Find :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

28. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1| + |x-2|) \, dx$$

29. Find :

$$\int \frac{x}{(x^2+1)(x-1)} \, dx$$

30. Solve the following Linear Programming Problem graphically:

Minimise $z = 6x + 7y$

subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

31. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that $y = 1$ when $x = 0$.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that $y = 0$ when $x = 1$.

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1, 2)$, $(1, 5)$ and $(3, 4)$.

33. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$\begin{aligned} x + y + z &= 6 \\ x + 2z &= 7 \\ 3x + y + z &= 12 \end{aligned}$$

34. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both a and b are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

35. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

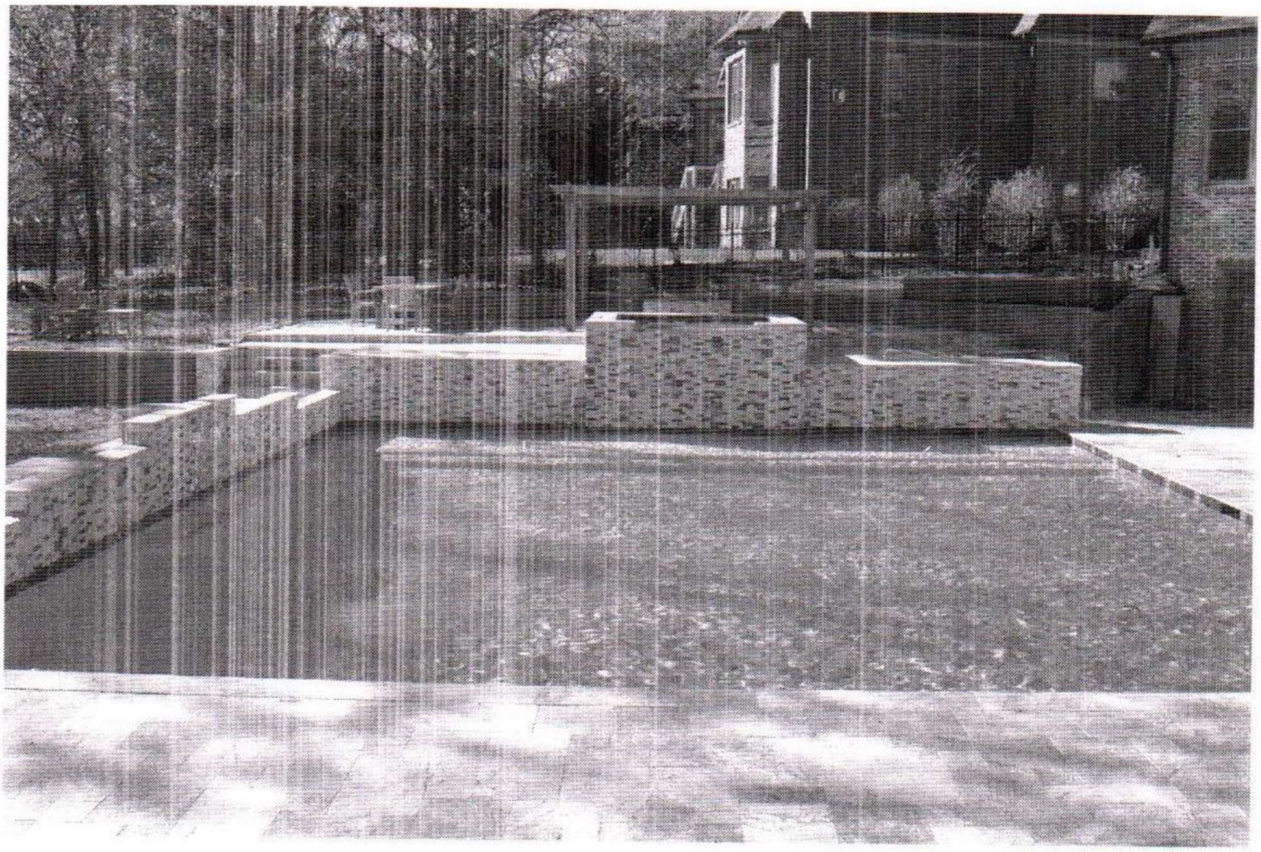
(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.



Suppose the side of the square plot is x metres and depth is h metres.
On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

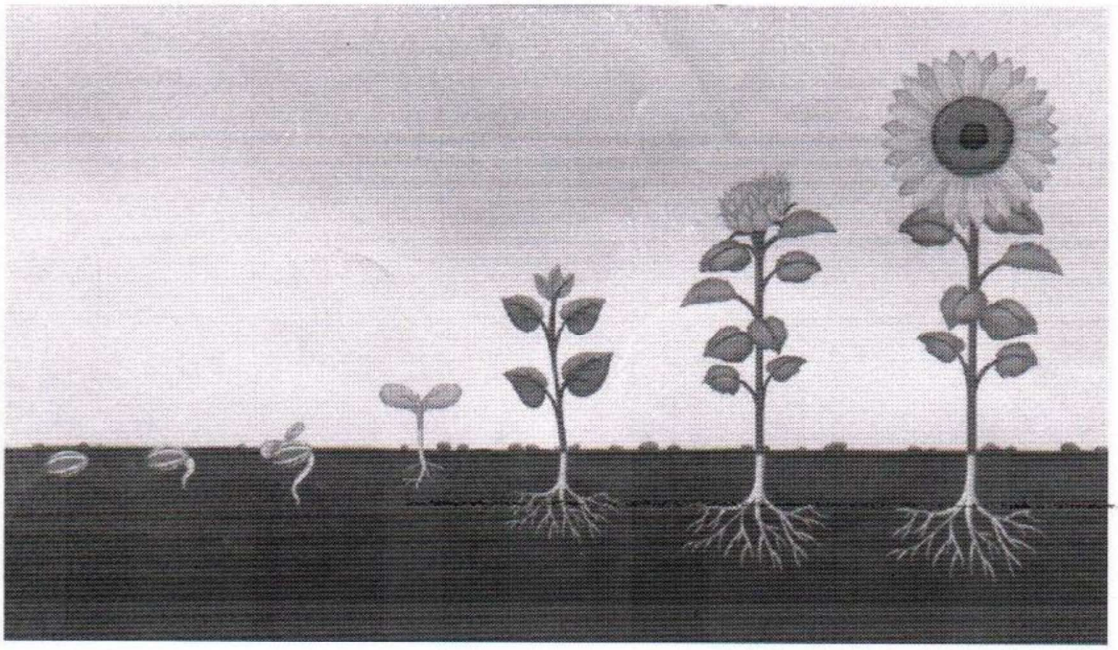
Case Study – 2

37. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \ 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

Case Study – 3

38. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

2023 Compt.

Series ϵFGHE/C

Set-5

प्रश्न-पत्र कोड
Q.P. Code **65(B)**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***

*Time allowed : **3** hours* *Maximum Marks : **80***

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The domain of the function $\sin^{-1}(2x)$ is :

(a) $[-1, 1]$ (b) $[0, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

2. If the matrix $\begin{bmatrix} 0 & 3 & 5 \\ k+1 & 0 & 4 \\ -5 & k & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value of k is :

(a) 4 (b) 2

(c) −2 (d) −4

3. If $A = \begin{bmatrix} 4 & -3 \\ 9 & -3 \end{bmatrix}$, then $|A|$ is equal to :

(a) 15 (b) −42

(c) 0 (d) 25

4. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$, then the value of α is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

5. If A is a 2×3 matrix and B is another matrix such that both $A'B$ and BA' are defined, then order of B is :

(a) 3×2 (b) 2×3

(c) 3×3 (d) 2×2

6. If $y = 3 \log \sqrt{\sin x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is :

(a) 3 (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

7. An angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine, is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

8. The value of k for which the function f, given by

$$f(x) = \begin{cases} kx + 1, & \text{if } \quad x \leq \pi \\ \cos x, & \text{if } \quad x > \pi \end{cases}$$

is continuous at $x = \pi$, is :

(a) $\frac{2}{\pi}$ (b) $-\frac{\pi}{2}$

(c) $-\frac{2}{\pi}$ (d) $\frac{\pi}{2}$

9. If $y = \sin^{-1}(2x\sqrt{1-x^2})$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ is :

(a) 2 (b) $\frac{2}{\sqrt{1-x^2}}$

(c) $\frac{-2}{\sqrt{1-x^2}}$ (d) $\sqrt{1-x^2}$

10. The rate of change of the volume of a spherical bubble with respect to its radius r at r = 3 cm is :

(a) $24\pi \text{ cm}^3/\text{cm}$ (b) $36\pi \text{ cm}^2/\text{cm}$

(c) $36\pi \text{ cm}^3/\text{cm}$ (d) $24\pi \text{ cm}^2/\text{cm}$

11. $\int \cot^2 x \, dx$ equals :

(a) $-\cot x + x + C$ (b) $-\cot x - x + C$

(c) $\cot x - x + C$ (d) $\cot x + x + C$

12. The value of $\int_{-1}^{1} \frac{dx}{5 + 2x + x^2}$ is :

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

13. The sum of the order and the degree of the differential equation $3x^2\left(\frac{d^2y}{dx^2}\right)^3 + 5\frac{d^3y}{dx^3} + 9 = 0$ is :

(a) 5 (b) 6

(c) 4 (d) 2

14. A particular solution of the differential equation $dy = y \tan x \, dx$; $y = 1$ when $x = 0$, is :

(a) $y = 0$ (b) $y = 1 + \sec x$

(c) $y = \sec x$ (d) $y \cos x = 0$

15. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then the value of θ is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

16. The position vectors of two points A and B are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively. The position vector of a point C which divides AB externally in the ratio 1 : 2 is :

(a) $-3\vec{a} - 5\vec{b}$ (b) $-7\vec{b}$

(c) $\frac{1}{3}(5\vec{a} - \vec{b})$ (d) $(3\vec{a} + 5\vec{b})$

17. The value of λ for which the points A, B and C having position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k})$, $(\hat{i} + \lambda\hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are collinear, is :

(a) 4 (b) 1

(c) 3 (d) 2

18. The direction ratios of a line whose Cartesian equations are $3x - 3 = 2y + 1 = 5 - 6z$, are :

(a) 2, 3, –1 (b) 3, –2, 1

(c) 2, 1, –3 (d) 3, 2, –1

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The vector equation of a line passing through the points (1, 2, 3) and (3, 0, 2) is $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} - \hat{k})$.

Reason (R) : Equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

20. *Assertion (A) :* Given two independent events A and B such that P(A) = 0·3 and P(B) = 0·6, then P(A and not B) = 0·12.

Reason (R) : For two independent events A and B, P(A and B) = P(A) . P(B).

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Let the relation R be given as

$R = \{(x, y) : x, y \in N \text{ and } x + 3y = 12\}$. Find the domain and range of R.

OR

(b) Show that the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$ is neither one-one nor onto.

22. Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

23. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

24. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = (3\hat{i} + \hat{k})$ and the other is perpendicular to $\vec{b}$.

25. (a) Find the angle between the pair of lines given by

$$\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z+4}{2}; \frac{x-5}{-3} = \frac{y+2}{2} = \frac{z}{6}.$$

OR

(b) If the lines $\frac{x-1}{-3} = \frac{2y-2}{4k} = \frac{3-z}{-2}$ and $\frac{x-1}{3k} = \frac{3y-1}{6} = \frac{z-6}{-5}$ are perpendicular to each other, find the value of k.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find the intervals in which the function given by

$f(x) = \sin 3x,\ x \in \left[0, \frac{\pi}{2}\right]$ is (a) increasing (b) decreasing.

27. Find :

$$\int \frac{x-5}{(x-2)^4} e^x \, dx$$

28. (a) Find :

$$\int \frac{2}{(1-x)(1+x^2)}\, dx$$

OR

(b) Find :

$$\int \sqrt{3-2x-x^2}\, dx$$

29. Evaluate :

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$$

30. (a) Solve the differential equation $x\frac{dy}{dx} = y - x \tan\frac{y}{x}$.

OR

(b) Find the particular solution of the differential equation $x\frac{dy}{dx} + y = x \cos x + \sin x$; given $y\left(\frac{\pi}{2}\right) = 1$.

31. (a) Let A and B be the events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. Find whether A and B are (i) mutually exclusive (ii) independent.

OR

(b) Find the mean of the number of tails in two tosses of a coin.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Let R be a relation in $\mathbb{R}$, the set of all real numbers, defined by $R = \{(a, b) : a \leq b^3\}$.

Show that R is neither reflexive, nor symmetric and nor transitive.

OR

(b) Let set $A = \{1, 2, 3, ..., 10\}$ and R be a relation in $A \times A$, defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all (a, b) and $(c, d) \in A \times A$. Prove that R is an equivalence relation.

33. Using integration, find the area enclosed by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

34. While solving the linear programming problem Minimise and Maximise $Z = 3x + 9y$, subject to the constraints $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$ and $x \geq 0$, $y \geq 0$ graphically, the corner points of the feasible region ABCD are A(0, 10), B(5, 5), C(15, 15) and D(0, 20). Find the minimum value and the maximum value of Z along with the corresponding corner points.

35. (a) Show that the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} - \hat{j} + 2\hat{k})$ do not intersect.

OR

(b) Find the coordinates of the foot of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Hence, write the equation of this perpendicular line.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A professional typist having his shop in a busy market charges ₹ 200 for typing 8 English and 4 Hindi pages, while he charges ₹ 275 for typing 5 English and 10 Hindi pages.

Based on the above information, answer the following questions :

(i) If he charges ₹ x for one page of English and ₹ y for one page of Hindi, express the above as a pair of linear equations. *1*

(ii) Express the information in terms of matrix equation AX = B. *1*

(iii) (a) Find $|A|$. *2*

OR

(iii) (b) Find (adj A). *2*

Case Study – 2

37. Ravindra started to run a small factory of manufacturing LED bulbs. He can sell x bulbs at a price of ₹ $(300 - x)$ each. The cost price of x bulbs is ₹ $(2x^2 - 60x + 18)$.

Based on the above information, answer the following questions :

(i) Find the profit function $P(x)$ for selling x bulbs. *1*

(ii) What is $\frac{d}{dx}[P(x)]$? *1*

(iii) (a) How many bulbs should he sell to earn maximum profit ? *2*

OR

(iii) (b) How many bulbs is he selling if he is incurring a loss of ₹ 18 ? *2*

Case Study – 3

38. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

Based on the above information, answer the following questions :

(i) What is the probability of a randomly chosen seed to germinate ? *2*

(ii) What is the probability that the randomly selected seed is of type A_1, given that it germinates ? *2*

2023 Annual

Series ϵFGHE

Set-5

प्रश्न-पत्र कोड
Q.P. Code **65(B)**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
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गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
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- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The domain of the function $\cos^{-1} x$ is :

(a) $[0, \pi]$ (b) $(-1, 1)$

(c) $[-1, 1]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. In a 3×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{1}{2}|-3i + j|$, the element a_{31} is :

(a) -4 (b) 5

(c) 4 (d) 8

3. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then $2x - y$ is equal to :

(a) 3 (b) 13

(c) -3 (d) 0

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then A^{-1} is given by :

(a) $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$

5. In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, M_{23} is :

(where M_{ij} denotes the minor of element a_{ij})

(a) 7 (b) -13

(c) 13 (d) -7

6. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to :

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 1 (d) $\frac{1}{2}$

7. The value of k for which the function f given by $f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 5, & \text{if } x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$, is :

(a) 6 (b) 5

(c) $\dfrac{5}{2}$ (d) 10

8. $\displaystyle\int \frac{dx}{\sin^2 2x \cdot \cos^2 2x}$ equals :

(a) $\dfrac{1}{2}$ [tan 2x + cot 2x] + C (b) tan 2x – cot 2x + C

(c) $\dfrac{1}{2}$ [tan 2x – cot 2x] + C (d) $\dfrac{1}{2}$ [cot 2x – tan 2x] + C

9. $\displaystyle\int_{-\pi/4}^{\pi/4} \sin^3 x \, dx$ equals :

(a) $2\displaystyle\int_{0}^{\pi/4} \sin^3 x \, dx$ (b) 0

(c) 1 (d) $\displaystyle\int_{0}^{\pi/4} \sin^3 x \, dx$

10. The general solution of the differential equation $\dfrac{dy}{dx} = e^{x-y}$ is :

(a) $e^x + e^y = C$ (b) $e^x - e^{-y} = C$

(c) $-e^x - e^y = C$ (d) $e^x - e^y = C$

11. The sum of the order and the degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^2 + 3x\left(\dfrac{d^2y}{dx^2}\right)^4 = \log x$, is :

(a) 5 (b) 6

(c) 7 (d) 4

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. $\vec{a} + \vec{b}$ is a unit vector, if :

(a) $\theta = \dfrac{\pi}{3}$ (b) $\theta = \dfrac{\pi}{4}$

(c) $\theta = \dfrac{\pi}{2}$ (d) $\theta = \dfrac{2\pi}{3}$

13. If $(2\hat{i} + 6\hat{j} - 22\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$, then $\lambda - \mu$ is equal to :

(a) -8 (b) -14

(c) 14 (d) 8

14. If the position vectors of two points A and B are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $-\hat{i} - 2\hat{j} + \hat{k}$ respectively, then the direction cosines of the vector $\overrightarrow{BA}$ are :

(a) $\dfrac{2}{6}, -\dfrac{4}{6}, -\dfrac{4}{6}$ (b) $\dfrac{1}{3}, \dfrac{2}{3}, -\dfrac{2}{3}$

(c) $\dfrac{1}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}, -\dfrac{2}{\sqrt{3}}$ (d) $-\dfrac{1}{3}, -\dfrac{2}{3}, \dfrac{2}{3}$

15. The value of λ for which the lines $\dfrac{x-5}{7} = \dfrac{2-y}{5} = \dfrac{z}{1}$ and $\dfrac{x}{1} = \dfrac{2y-1}{\lambda} = \dfrac{z}{3}$ are at right angles, is :

(a) 2 (b) 4

(c) -4 (d) -2

16. The solution set of the inequation $2x + y \geq 5$ is :

(a) half plane that contains the origin

(b) open half plane not containing the origin and not containing the points on the line $2x + y = 5$.

(c) whole xy-plane except the points lying on the line $2x + y = 5$.

(d) open half plane not containing the origin, but containing the points on the line $2x + y = 5$.

17. The minimum value of $z = 3x + 8y$ subject to the constraints $x \leq 20$, $y \geq 10$ and $x \geq 0$, $y \geq 0$ is :

(a) 80 (b) 140

(c) 0 (d) 60

18. Two events A and B will be independent, if :

(a) A and B are mutually exclusive

(b) $P(A) = P(B)$

(c) $P(A'B') = [1 - P(A)]\ [1 - P(B)]$

(d) $P(A) + P(B) = 1$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* Matrix $A = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & -2 \\ -5 & 2 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

Reason (R) : If $A' = -A$, then A is a skew-symmetric matrix.

20. *Assertion (A) :* The vector equation of a line passing through the points A(–1, 0, 2) and B(3, 4, 6) is $\vec{r} = -\hat{i} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$.

Reason (R) : The equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$, is $\vec{r} = \vec{a} + \lambda \vec{b}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

OR

(b) Check the injectivity and surjectivity of the function $f : N \to N$, given by $f(x) = x^3$.

22. If $\cos y = x \cos(a + y)$, and $\cos a \neq \pm 1$, prove that
$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}.$$

23. Find the projection of the vector $7\hat{i} - \hat{j} + 8\hat{k}$ on the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

24. (a) In a parallelogram ABCD, the sides AB and AD are represented by the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$ respectively. Find the unit vector parallel to its diagonal $\overrightarrow{AC}$.

OR

(b) Find the angle between the pair of lines given by

$\vec{r} = \hat{i} + 2\hat{j} - 2\hat{k} + \lambda(\hat{i} - 2\hat{j} - 2\hat{k})$ and

$\vec{r} = 3\hat{i} - 5\hat{j} + \hat{k} + \mu(3\hat{i} + 2\hat{j} - 6\hat{k})$.

25. The radius of an air bubble is increasing at the rate of 0·5 cm/s. At what rate is the surface area of the bubble increasing when the radius is 1·5 cm ?

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{dx}{1 + \cot x}$$

27. (a) Evaluate :

$$\int_{\pi/2}^{\pi} e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$$

OR

(b) Evaluate :

$$\int_{0}^{\pi/4} \log\,(1 + \tan x)\, dx$$

28. Find :

$$\int \frac{x}{(x^2+1)(x-1)}\, dx$$

29. (a) Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$, $(x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$.

OR

(b) Solve the differential equation $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$.

30. The objective function $z = 4x + 3y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are A(0, 700), B(100, 700), C(200, 600) and D(400, 200). Find the point at which z is maximum and the point at which z is minimum. Also, find the corresponding maximum and minimum values of z.

31. (a) A man is known to speak the truth 3 out of 5 times. He throws a pair of different coins and reports that he got a pair of heads. Find the probability that a pair of heads actually occurs.

OR

(b) From a lot of 10 bulbs which includes 2 defectives, a sample of 2 bulbs is drawn at random without replacement. Find the probability distribution of the number of defective bulbs. Hence, find the mean.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) A relation R in the set A = {5, 6, 7, 8, 9} is given by R = {(x, y) : |x – y| is divisible by 2}. Write R in roster form and prove that R is an equivalence relation. Also, find the elements related to element 7.

OR

(b) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$ be two sets. Prove that the function $f : A \to B$ given by $f(x) = \left(\frac{x-2}{x-3}\right)$ is onto. Is the function f one-one ? Justify your answer.

33. Using matrices, solve the following system of linear equations :

$$x - y + 2z = 7 \; ; \; 3x + 4y - 5z = -5 \; ; \; 2x - y + 3z = 12$$

34. Find the area of the region bounded by the curve $y = \sqrt{x}$, the line $x = 2y + 3$ and the x-axis, using integration.

35. (a) Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and}$$

$$\vec{r} = 3\hat{i} - 3\hat{j} - 5\hat{k} + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

OR

(b) Find the foot of the perpendicular drawn from the point (2, 3, – 8) to the line $\frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$. Also, find the perpendicular distance of the given line from the given point.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A balloon is being inflated with the help of an air pump, and it remains spherical. Its radius, the surface area and the volume of air in it are all increasing.

Based on the above, answer the following questions :

(i) Are the quantities : radius, surface area and volume of the spherical balloon changing at the same rate or different rates, when air is filled in it ? *1*

(ii) Write the expressions for the surface area (S) and the volume (V) of the balloon at any time 't' in terms of radius 'r' at that instant. *1*

(iii) (a) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the surface area (S) of the balloon is increasing. *2*

OR

(iii) (b) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the volume (V) of the spherical balloon is increasing. *2*

Case Study – 2

37. A fighter-jet of the enemy is flying along the parabolic path $4y = x^2$. A soldier is located at the point (0, 5) and is aiming to shoot down the jet when it is nearest to him.

Based on the above, answer the following questions :

(i) Let (x, y) be the position of the jet at any instant. Express the distance between the soldier and the jet as the function f(x). *1*

(ii) Taking $S = [f(x)]^2$, find $\frac{dS}{dx}$. *1*

(iii) (a) What will be the position of the jet when the soldier shoots it down ? *2*

OR

(iii) (b) What will be the distance between the soldier and the jet at the instant when he shoots it down ? *2*

Case Study – 3

38. Read the following passage and answer the questions given below :

There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box.

Based on the above, answer the following questions :

(i) What is the probability that the number on the drawn card is greater than 4 ? *2*

(ii) If it is known that the number on the drawn card is greater than 4, then what is the probability that it is an even number ? *2*

2023 Annual

Series EF1GH/1

SET~1

प्रश्न–पत्र कोड
Q.P. Code 65/1/1

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2

(c) 3 (d) -3

2. If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals :

(a) 4 (b) 2

(c) 8 (d) $\frac{1}{32}$

3. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only

(c) 64 (d) 8 or -8

4. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5. If $\frac{d}{dx}(f(x)) = \log x$, then f(x) equals :

(a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$

(c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$

6. $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

7. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5 (b) 2

(c) 3 (d) 4

8. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3 (b) -3

(c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

9. The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is :

(a) 2 (b) 0

(c) 1 (d) -1

10. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

11. Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are :

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

12. If $P\left(\frac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) 0·6 (b) 0·3

(c) 0·06 (d) 0·4

13. The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is :

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$

(c) 11 (d) $\frac{11}{4}$

14. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are :

(a) $\pm\sqrt{7}$ (b) 0

(c) ± 5 (d) 25

15. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$

16. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) $\{0\}$ (b) $(0, \infty)$

(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

17. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

18. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2 (b) 3

(c) 4 (d) 5

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The range of the function $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

20. *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

OR

(b) Evaluate :

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

22. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

23. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

24. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

25. Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Evaluate :

$$\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2 \cos x)]\, dx.$$

27. Find :

$$\int \frac{1}{\sqrt{x}(\sqrt{x} + 1)(\sqrt{x} + 2)}\, dx$$

28. (a) Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \,.\, y = \tan x \,.\, \sec^2 x$, given that $y(0) = 0$.

OR

(b) Solve the differential equation given by
$x\,dy - y\,dx - \sqrt{x^2 + y^2}\,dx = 0$.

29. Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$,

subject to the constraints

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0, y \geq 0.$$

30. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

31. (a) Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

OR

(b) Find :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)} \, dx$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. A relation R is defined on a set of real numbers $\mathbb{R}$ as

$R = \{(x, y) : x . y \text{ is an irrational number}\}$.

Check whether R is reflexive, symmetric and transitive or not.

33. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method :

$x + 2y + 3z = 6$

$2x - y + z = 2$

$3x + 2y - 2z = 3$

34. (a) Find the vector and the Cartesian equations of a line passing through the point $(1, 2, -4)$ and parallel to the line joining the points $A(3, 3, -5)$ and $B(1, 0, -11)$. Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

35. Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.

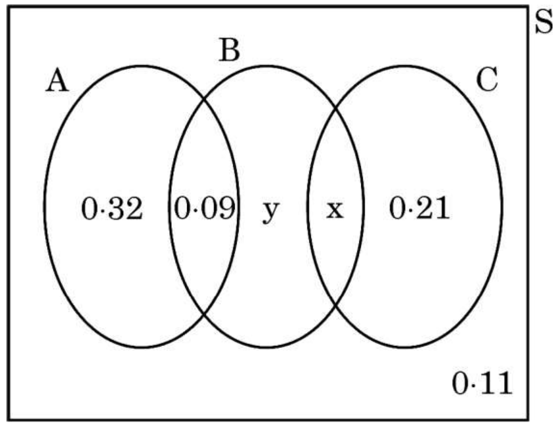
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x. *1*

(ii) Find the value of y. *1*

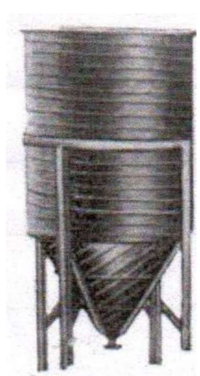
(iii) (a) Find $P\left(\frac{C}{B}\right)$. *2*

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. *2*

Case Study – 2

37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is $45°$.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r. *1*

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. *1*

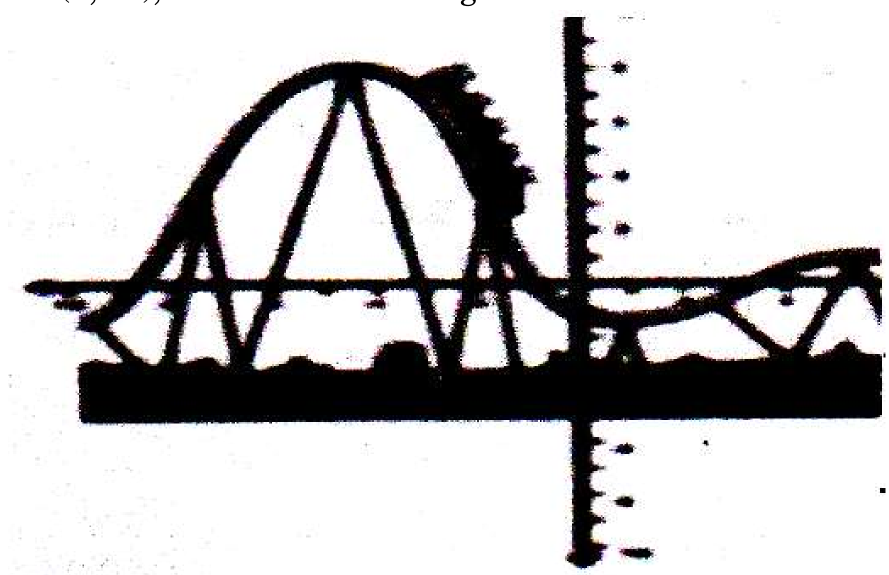
(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. *2*

OR

(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. *2*

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of 'a'. *2*

(ii) Find $f''(x)$ at $x = 1$. *2*

2023 Annual

Series EF1GH/1 **SET~2**

प्रश्न–पत्र कोड
Q.P. Code **65/1/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए** 15 **मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में** 10.15 **बजे किया जाएगा ।** 10.15 **बजे से** 10.30 **बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

*(i) This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

*(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

*(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

*(vii) In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

*(ix) Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. $\int \frac{1+\tan x}{1-\tan x} dx$ is equal to :

(a) $\sec^2\left(\frac{\pi}{4}+x\right)+C$ (b) $\sec^2\left(\frac{\pi}{4}-x\right)+C$

(c) $\log\left|\sec\left(\frac{\pi}{4}+x\right)\right|+C$ (d) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|+C$

2. $\int_0^{\frac{\pi}{6}} \sec^2(x-\frac{\pi}{6})\, dx$ is equal to :

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

3. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5 (b) 2

(c) 3 (d) 4

4. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3 (b) -3

(c) $-\dfrac{17}{3}$ (d) $\dfrac{17}{3}$

5. If the vector $\hat{i} - b\hat{j} + \hat{k}$ is equally inclined to the coordinate axes, then the value of b is :

(a) -1 (b) 1

(c) $-\sqrt{3}$ (d) $-\dfrac{1}{\sqrt{3}}$

6. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

7. Direction cosines of a line perpendicular to both x-axis and z-axis are :

(a) 1, 0, 1 (b) 1, 1, 1

(c) 0, 0, 1 (d) 0, 1, 0

8. If $P\left(\dfrac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\dfrac{B}{A}\right)$ is equal to :

(a) 0·6 (b) 0·3

(c) 0·06 (d) 0·4

9. For what value of k may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \le 0 \\ \cos x, & x > 0 \end{cases}$

become continuous ?

(a) 0 (b) 1

(c) $-\dfrac{1}{2}$ (d) No value

10. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2I$, then the value(s) x is/are :

(a) $\pm\sqrt{7}$ (b) 0

(c) ± 5 (d) 25

11. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$

(b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$

(d) $y = 2 \log x + \frac{x^2}{2} + C$

12. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) $\{0\}$

(b) $(0, \infty)$

(c) $(-\infty, 0)$

(d) $(-\infty, \infty)$

13. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

14. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2

(b) 3

(c) 4

(d) 5

15. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2

(b) 2

(c) 3

(d) -3

16. If $\left|\frac{A^{-1}}{2}\right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is :

(a) $\frac{1}{8}$

(b) 8

(c) 2

(d) $\frac{1}{2}$

17. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) – 8 only

(c) 64 (d) 8 or – 8

18. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

20. *Assertion (A) :* The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason (R) : Number of onto functions from a set containing m elements to a set containing n elements is n^m.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

22. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

23. Find the sub-intervals in which $f(x) = \log(2 + x) - \frac{x}{2 + x}$, $x > -2$ is increasing or decreasing.

24. (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

OR

(b) Evaluate :

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$$

25. For two non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find the general solution of the differential equation :

$$\frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y} - 1\right)}{1 + e^{x/y}}.$$

OR

(b) Find the particular solution of the differential equation $\frac{dy}{dx} + \cot x \cdot y = \cos^2 x$, given that when $x = \frac{\pi}{2}$, $y = 0$.

27. Solve the following linear programming problem graphically :

Maximize $P = 100x + 5y$

subject to the constraints

$$x + y \leq 300,$$
$$3x + y \leq 600,$$
$$y \leq x + 200,$$
$$x, y \geq 0.$$

28. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

29. (a) Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

OR

(b) Find :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)} \, dx$$

30. Evaluate :

$$\int_{0}^{\frac{\pi}{2}} |\sin x - \cos x| \, dx$$

31. Find :

$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} \, dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

33. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$, using integration.

34. A relation R is defined on a set of real numbers $\mathbb{R}$ as

$$R = \{(x, y) : x \cdot y \text{ is an irrational number}\}.$$

Check whether R is reflexive, symmetric and transitive or not.

35. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method :

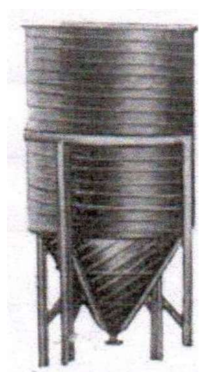
$$x + 2y + 3z = 6$$
$$2x - y + z = 2$$
$$3x + 2y - 2z = 3$$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is $45°$.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r. *1*

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. *1*

(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. *2*

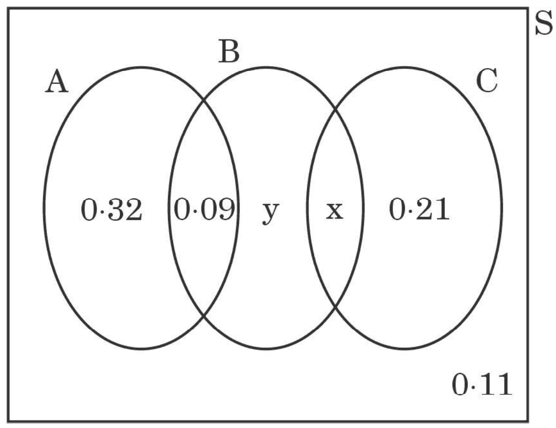
OR

(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. *2*

Case Study – 2

37. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x. *1*

(ii) Find the value of y. *1*

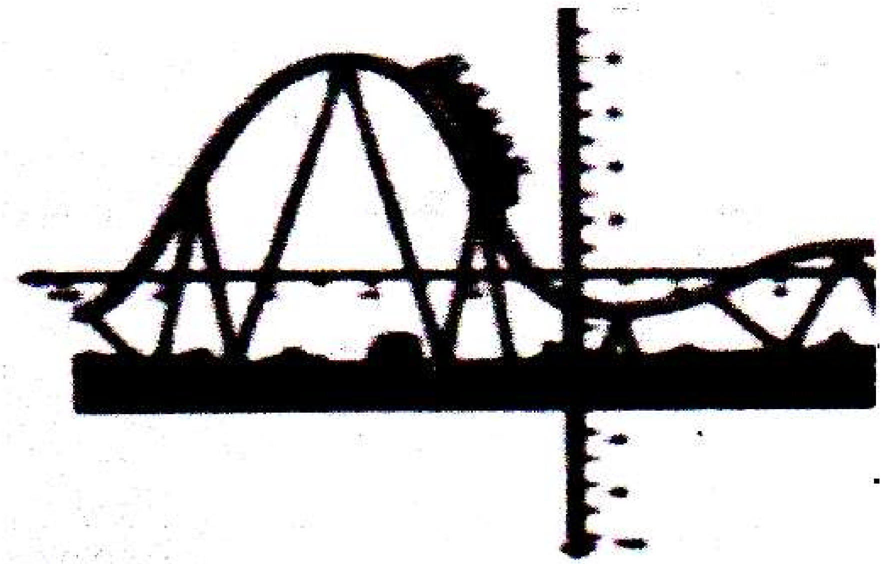
(iii) (a) Find $P\left(\frac{C}{B}\right)$. *2*

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. *2*

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of 'a'. *2*

(ii) Find $f''(x)$ at $x = 1$. *2*

2023 Annual

Series EF1GH/1 **SET~3**

प्रश्न–पत्र कोड
Q.P. Code **65/1/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
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(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
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(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

2. The direction ratios of a line parallel to z-axis are :

(a) $< 1, 1, 0 >$ (b) $< 1, 1, 1 >$

(c) $< 0, 0, 0 >$ (d) $< 0, 0, 1 >$

3. Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly once is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{9}$ (d) $\frac{1}{9}$

4. The function $f(x) = |x| - x$ is :

(a) continuous but not differentiable at $x = 0$.

(b) continuous and differentiable at $x = 0$.

(c) neither continuous nor differentiable at $x = 0$.

(d) differentiable but not continuous at $x = 0$.

5. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are :

(a) $\pm \sqrt{7}$ (b) 0

(c) ± 5 (d) 25

6. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2 \log x + \frac{x^3}{3} + C$

(c) $y = \frac{x^2}{2} + C$ (d) $y = 2 \log x + \frac{x^2}{2} + C$

7. If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) $\{0\}$ (b) $(0, \infty)$

(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

8. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then :

(a) z is maximum at (2, 72), minimum at (15, 20)

(b) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(d) z is maximum at (40, 15), minimum at (2, 72)

9. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is :

(a) 2 (b) 3

(c) 4 (d) 5

10. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is :

(a) -2 (b) 2

(c) 3 (d) -3

11. If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is :

(a) 6 (b) -6

(c) 0 (d) -7

12. Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only

(c) 64 (d) 8 or -8

13. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to :

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

14. The primitive of $\dfrac{2}{1 + \cos 2x}$ is

(a) $\sec^2 x$ (b) $2 \sec^2 x \tan x$

(c) $\tan x$ (d) $-\cot x$

15. $\displaystyle\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to :

(a) $\dfrac{1}{\sqrt{3}}$ (b) $-\dfrac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

16. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(a) 5 (b) 2

(c) 3 (d) 4

17. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3 (b) -3

(c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

18. For what value of λ, the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$?

(a) -1 (b) 1

(c) 0 (d) 3

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

20. *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum.

22. (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

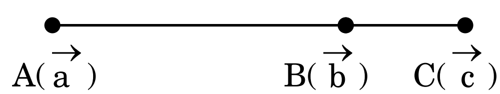
OR

(b) Evaluate :

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}\left(\cos \frac{3\pi}{4}\right) + \tan^{-1}(1)$$

23. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.

24. (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

OR

(b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

25. If $x = \sqrt{a^{\tan^{-1} t}}$, $y = \sqrt{a^{\cot^{-1} t}}$, then show that $x\frac{dy}{dx} + y = 0$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Evaluate :

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx$$

OR

(b) Find :

$$\int \frac{1}{\cos(x-a)\ \cos(x-b)}\ dx$$

27. Evaluate :

$$\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]\ dx$$

28. Find :

$$\int \frac{x^2}{(x^2+4)(x^2+9)} dx$$

29. (a) Find the general solution of the differential equation : $\frac{dy}{dx} - \frac{2y}{x} = \sin\frac{1}{x}$.

OR

(b) Find the particular solution of the differential equation : $\frac{dy}{dx} = \sin(x+y) + \sin(x-y)$, given that when $x = \frac{\pi}{4}$, $y = 0$.

30. Solve the following linear programming problem graphically :

Maximize $z = 600x + 400y$

subject to the constraints :

$$x + 2y \le 12,$$
$$2x + y \le 12,$$
$$x + 1{\cdot}25y \ge 5,$$
$$x, y \ge 0$$

31. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \le X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

OR

(b) Solve the following system of equations by matrix method :

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

33. (a) Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines.

OR

(b) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

34. Find the area of the region bounded by the lines $y = 4x + 5$, $x + y = 5$ and $4y = x + 5$, using integration.

35. A relation R is defined on a set of real numbers $\mathbb{R}$ as

$$R = \{(x, y) : x \cdot y \text{ is an irrational number}\}.$$

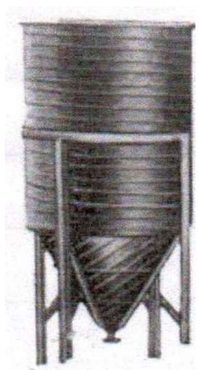
Check whether R is reflexive, symmetric and transitive or not.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r. *1*

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. *1*

(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. *2*

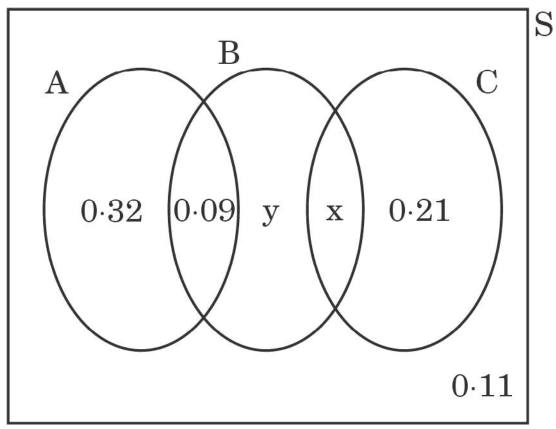
OR

(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm. *2*

Case Study – 2

37. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x. *1*

(ii) Find the value of y. *1*

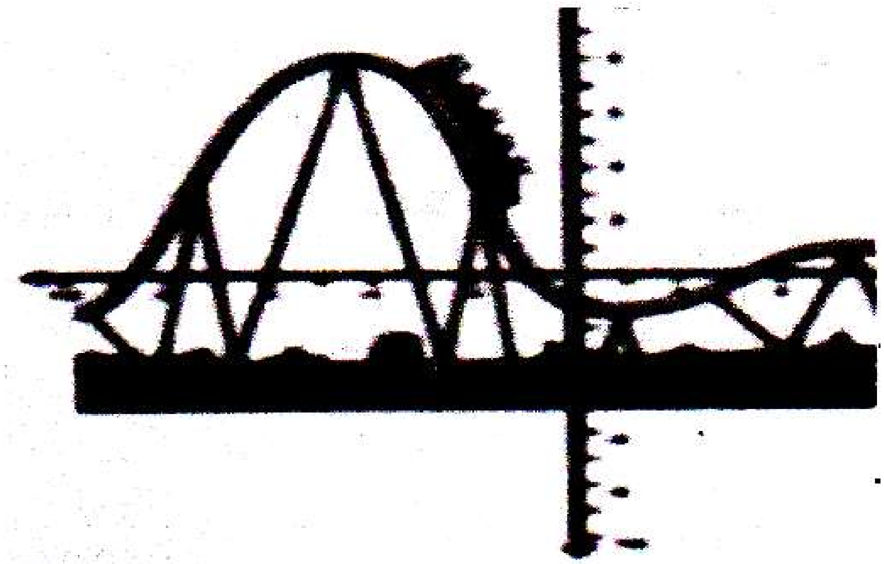
(iii) (a) Find $P\left(\frac{C}{B}\right)$. *2*

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. *2*

Case Study – 3

38. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :



(i) Find the value of 'a'. *2*

(ii) Find $f''(x)$ at $x = 1$. *2*

2023 Annual

Series EF1GH/2 **SET~1**

प्रश्न-पत्र कोड
Q.P. Code **65/2/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This Question Paper contains* ***38*** *questions.* ***All*** *questions are compulsory.*

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A*** *– Question Nos.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and Question Nos.* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B*** *– Question Nos.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions of* ***2*** *marks each.*

(v) *In* ***Section C*** *– Question Nos.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D*** *– Question Nos.* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E*** *– Question Nos.* ***36*** *to* ***38*** *are source based/case based/passage based/integrated units of assessment questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section* ***B****,* ***3*** *questions in Section* ***C****,* ***2*** *questions in Section* ***D*** *and* ***2*** *questions in Section* ***E****.*

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A
(Multiple Choice Questions)
Each question carries 1 mark.

1. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to **1**

(A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

2. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to **1**

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

3. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is **1**

(A) $\mathbb{R}$

(B) $\{0\}$

(C) $\{4\}$

(D) $\mathbb{R} - \{4\}$

4. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1

(B) –1

(C) 2

(D) 0

5. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to **1**

(A) $a + b$

(B) $\frac{ax^2}{2} + bx$

(C) $\frac{ax^2}{2} + bx + c$

(D) b

6. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **1**

(A) 2

(B) 1

(C) not defined

(D) 0

7. The integrating factor of the differential equation

$$(1-y^2)\frac{dx}{dy} + yx = ay,\ (-1 < y < 1) \text{ is}$$ **1**

(A) $\frac{1}{y^2-1}$ (B) $\frac{1}{\sqrt{y^2-1}}$

(C) $\frac{1}{1-y^2}$ (D) $\frac{1}{\sqrt{1-y^2}}$

8. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is **1**

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

9. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is **1**

(A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$

(C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

10. Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is **1**

(A) 0 (B) 12

(C) $\frac{12}{\sqrt{13}}$ (D) $\frac{-12}{\sqrt{13}}$

11. Equation of a line passing through point (1, 1, 1) and parallel to z-axis is **1**

(A) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (B) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

(C) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (D) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

12. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is : **1**

(A) $\frac{1}{9}$ (B) $\frac{4}{9}$

(C) $\frac{1}{18}$ (D) $\frac{1}{2}$

13. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is **1**

(A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$ (B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$ (D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

14. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to **1**

(A) $2\Delta^2$ (B) $4\Delta^2$

(C) 2Δ (D) 4Δ

15. The function $f(x) = x|x|$ is **1**

(A) continuous and differentiable at $x = 0$.

(B) continuous but not differentiable at $x = 0$.

(C) differentiable but not continuous at $x = 0$.

(D) neither differentiable nor continuous at $x = 0$.

16. If $\tan\left(\frac{x + y}{x - y}\right) = k$, then $\frac{dy}{dx}$ is equal to **1**

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

17. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? **1**

(A) $a = 9, b = 1$ (B) $a = 5, b = 2$

(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

18. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to **1**

(A) 40 (B) 96

(C) 120 (D) 136

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **1**

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

20. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **1**

Reason (R) : The sum of squares of the direction cosines of a line is 1.

SECTION – B

This section comprises Very Short Answer Type questions (VSA) of **2** marks each.

21. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

22. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a. **2**

23. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$. **2**

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. **2**

25. (a) If $y = \sqrt{ax + b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \le 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b. **2**

SECTION – C

This section comprises Short Answer type questions (SA) of **3** marks each.

26. (a) Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\, dx$. **3**

OR

(b) Find $\int \frac{dx}{\sqrt{\sin^3 x \cos(x - \alpha)}}$. **3**

27. Find $\int e^{\cot^{-1} x}\left(\frac{1-x+x^2}{1+x^2}\right) dx$. **3**

28. Evaluate $\displaystyle\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}\,dx$ **3**

29. (a) Find the general solution of the differential equation :

$(xy - x^2)\,dy = y^2\,dx$. **3**

OR

(b) Find the general solution of the differential equation :

$$(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$$ **3**

30. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **3**

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first. **3**

31. Solve the following linear programming problem graphically : **3**

Minimize : $Z = 5x + 10y$

subject to constraints : $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$,

$x \ge 0$, $y \ge 0$

SECTION – D

This section comprises Long Answer type questions (LA) of **5** marks each.

32. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$

$$2x - y - z = 2$$

$$-2y + z = 3$$ **5**

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **5**

33. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. **5**

OR

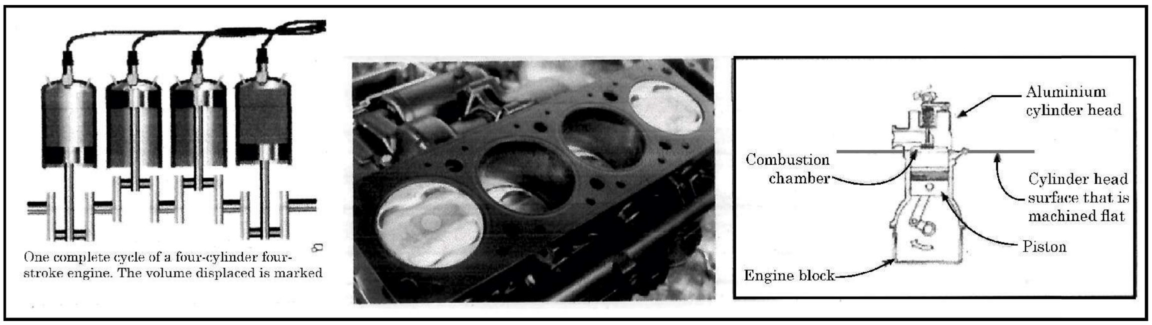
(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin. **5**

34. Using integration, find the area of region bounded by line $y = \sqrt{3}x$, the curve $y = \sqrt{4 - x^2}$ and y–axis in first quadrant. **5**

35. A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. **5**

This section comprises 3 source based/case-based/passage based/integrated units of assessment questions of **4** marks each.

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

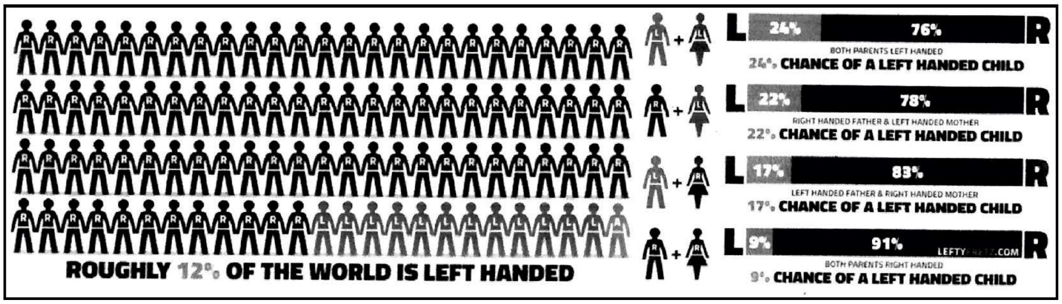
(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

37. आधुनिक अध्ययन यह सुझाते हैं कि दुनिया की आबादी में लगभग 12% लोग वामहस्तिक हैं ।



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$ **1**

(ii) Find $P(\overline{L}/A)$ **1**

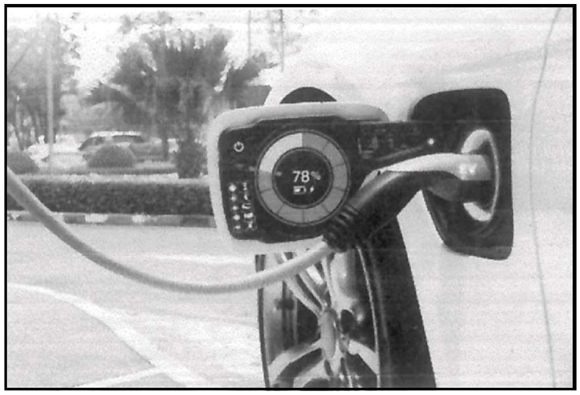
(iii) (a) Find $P(A/L)$ **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

38. विद्युत वाहनों का प्रयोग अंत में वायु प्रदूषण पर नियंत्रण कर लेगा ।

38. The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**

2023 Annual

Series EF1GH/2

SET~2

प्रश्न-पत्र कोड
Q.P. Code 65/2/2

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*निर्धारित समय : **3** घण्टे* — *अधिकतम अंक : **80***

*Time allowed : **3** hours* — *Maximum Marks : **80***

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This Question Paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Nos. **1** to **18** are Multiple Choice Questions (MCQs) and Question Nos. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Nos. **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C** – Question Nos. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Nos. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E** – Question Nos. **36** to **38** are source based/case based/passage based/integrated units of assessment questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, 2 questions in Section **D** and **2** questions in Section **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries 1 mark.

1. If $\frac{d}{dx} f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is **1**

(A) $x^2 + 3 \log |x| + 1$ (B) $x^2 + 3 \log |x|$

(C) $2 - \frac{3}{x^2}$ (D) $x^2 + 3 \log |x| - 4$

2. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **1**

(A) 2 (B) 1

(C) not defined (D) 0

3. The integrating factor of the differential equation

$(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is **1**

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$

(C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$

4. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is **1**

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

5. If in ΔABC, $\overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is **1**

(A) $2\vec{a} + 3\vec{b}$ (B) $2\vec{a} - 3\vec{b}$

(C) $3\vec{b} - 2\vec{a}$ (D) $-2\vec{a} - 3\vec{b}$

6. If $|\vec{a} \times \vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = -3$, then angle between $\vec{a}$ and $\vec{b}$ is **1**

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

7. Equation of line passing through origin and making 30°, 60° and 90° with x, y, z axes respectively is **1**

(A) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

(B) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

(C) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

(D) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

8. If A and B are two events such that P(A/B)=2 × P(B/A) and P(A) + P(B) = $\frac{2}{3}$, then P(B) is equal to **1**

(A) $\frac{2}{9}$

(B) $\frac{7}{9}$

(C) $\frac{4}{9}$

(D) $\frac{5}{9}$

9. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is : **1**

(A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$

(B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

(D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

10. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to **1**

(A) $2\Delta^2$

(B) $4\Delta^2$

(C) 2Δ

(D) 4Δ

11. The function $f(x) = x|x|$ is **1**

(A) continuous and differentiable at $x = 0$.

(B) continuous but not differentiable at $x = 0$.

(C) differentiable but not continuous at $x = 0$.

(D) neither differentiable nor continuous at $x = 0$.

12. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to **1**

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

13. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? **1**

(A) $a = 9, b = 1$ (B) $a = 5, b = 2$

(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

14. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to **1**

(A) 40 (B) 96

(C) 120 (D) 136

15. If A is a 2×3 matrix such that AB and AB' both are defined, then order of the matrix B is **1**

(A) 2×2 (B) 2×1

(C) 3×2 (D) 3×3

16. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to **1**

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

17. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a $\in$ A, then the set A is **1**

(A) $\mathbb{R}$ (B) $\{0\}$

(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

18. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1 (B) -1

(C) 2 (D) 0

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **1**

Reason (R) : The sum of squares of the direction cosines of a line is 1.

20. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **1**

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

SECTION – B

This section comprises of Very Short Answer Type (VSA) questions, each of **2** marks.

21. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$. **2**

22. (a) Evaluate $\sin^{-1}\left(\sin \dfrac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

23. If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line. **2**

24. (a) If $y = \sqrt{ax + b}$, prove that $y\left(\dfrac{d^2y}{dx^2}\right) + \left(\dfrac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \leq 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b. **2**

25. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius. **2**

SECTION – C

The section comprises Short Answer (SA) type questions of **3** marks each.

26. Evaluate $\displaystyle\int_{\log \sqrt{2}}^{\log \sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})} dx$ **3**

27. (a) Find the general solution of the differential equation :

$(xy - x^2)\,dy = y^2\,dx.$ **3**

OR

(b) Find the general solution of the differential equation :

$$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$ **3**

28. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **3**

OR

(b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first. **3**

29. Solve the following linear programming problem graphically :

Maximize : $Z = x + 2y$

subject to constraints : $x + 2y \geq 100,$

$2x - y \leq 0,$

$2x + y \leq 200,$

$x \geq 0, y \geq 0.$ **3**

30. (a) Evaluate $\int_{-1}^{1} |x^4 - x|\,dx.$ **3**

OR

(b) Find $\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}\,dx.$ **3**

31. Find $\int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$ **3**

SECTION – D

This section comprises Long Answer type (LA) questions of **5** marks each.

32. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. **5**

OR

(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin. **5**

33. Using Integration, find the area of triangle whose vertices are (–1, 1), (0, 5) and (3, 2). **5**

34. A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. **5**

35. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$x - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$ **5**

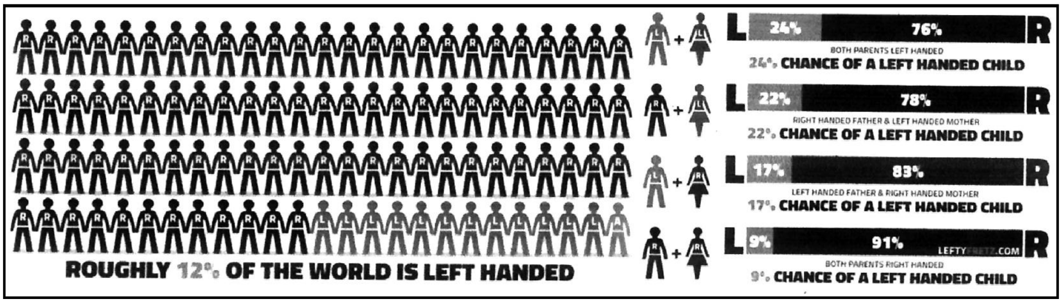
OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **5**

SECTION – E

This section comprises 3 source based case-based/passage based/integrated units of assessment questions of **4** marks each.

36. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$ **1**

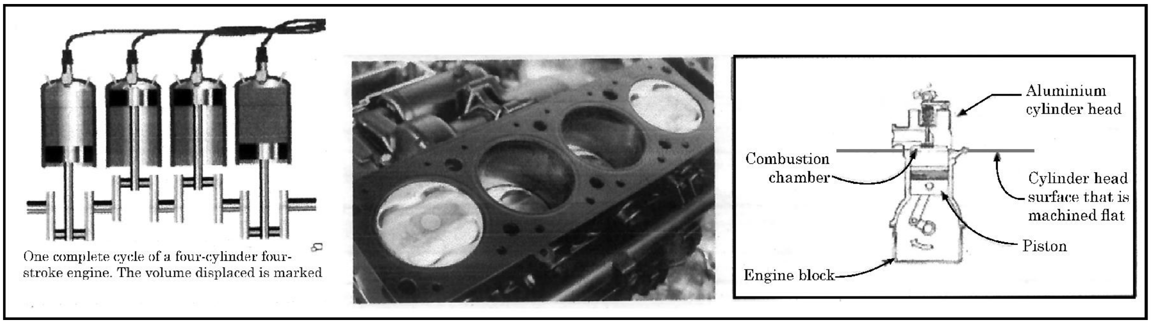
(ii) Find $P(\overline{L}/A)$ **1**

(iii) (a) Find $P(A/L)$ **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

37. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

2023 Annual

Series EF1GH/2

SET~3

प्रश्न-पत्र कोड
Q.P. Code **65/2/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This Question Paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A** – Question Nos. **1** to **18** are Multiple Choice Questions (MCQs) and Question Nos. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Nos. **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C** – Question Nos. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Nos. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E** – Question Nos. **36** to **38** are source based/case based/passage based/integrated units of assessment questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION – A
(Multiple Choice Questions)
Each question carries 1 mark.

1. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is **1**

(A) 9 (B) 3

(C) $\frac{1}{9}$ (D) $\frac{1}{3}$

2. The position vectors of three consecutive vertices of a parallelogram ABCD are A $(4\hat{i} + 2\hat{j} - 6\hat{k})$, B $(5\hat{i} - 3\hat{j} + \hat{k})$ and C $(12\hat{i} + 4\hat{j} + 5\hat{k})$. The position vector of D is given by **1**

(A) $-3\hat{i} - 5\hat{j} - 10\hat{k}$ (B) $21\hat{i} + 3\hat{j}$

(C) $11\hat{i} + 9\hat{j} - 2\hat{k}$ (D) $-11\hat{i} - 9\hat{j} + 2\hat{k}$

3. If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to **1**

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

4. If $\int_0^{2\pi} \cos^2 x \, dx = k \int_0^{\pi/2} \cos^2 x \, dx$, then the value of k is **1**

(A) 4 (B) 2

(C) 1 (D) 0

5. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to **1**

(A) $2\Delta^2$ (B) $4\Delta^2$

(C) 2Δ (D) 4Δ

6. The function $f(x) = x|x|$ is **1**

(A) continuous and differentiable at $x = 0$.

(B) continuous but not differentiable at $x = 0$.

(C) differentiable but not continuous at $x = 0$.

(D) neither differentiable nor continuous at $x = 0$.

7. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to **1**

(A) $\frac{-y}{x}$

(B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$

(D) $-\sec^2\left(\frac{y}{x}\right)$

8. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? **1**

(A) $a = 9, b = 1$

(B) $a = 5, b = 2$

(C) $a = 3, b = 5$

(D) $a = 5, b = 3$

9. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to **1**

(A) 40

(B) 96

(C) 120

(D) 136

10. Number of symmetric matrices of order 3×3 with each entry 1 or – 1 is **1**

(A) 512

(B) 64

(C) 8

(D) 4

11. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to **1**

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

12. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a $\in$ A, then the set A is **1**

(A) $\mathbb{R}$ (B) $\{0\}$

(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

13. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1 (B) –1

(C) 2 (D) 0

14. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to **1**

(A) a + b (B) $\frac{ax^2}{2} + bx$

(C) $\frac{ax^2}{2} + bx + c$ (D) b

15. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **1**

(A) 2 (B) 1

(C) not defined (D) 0

16. The integrating factor of the differential equation

$$(1 - y^2)\frac{dx}{dy} + yx = ay, \ (-1 < y < 1) \text{ is}$$ **1**

(A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$

(C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$

17. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is **1**

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

18. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is **1**

(A) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (B) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

(C) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ (D) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **1**

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

20. **Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **1**

Reason (R) : The sum of squares of the direction cosines of a line is 1.

SECTION – B

This section comprises of Very Short Answer Type (VSA) questions, each of **2** marks.

21. If points A, B and C have position vectors $2\hat{i}$, $\hat{j}$ and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle. **2**

22. (a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range. **2**

23. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal ? **2**

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. **2**

25. (a) If $y = \sqrt{ax+b}$, then prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **2**

OR

(b) If $f(x) = \begin{cases} ax + b & ; \ 0 < x \le 1 \\ 2x^2 - x & ; \ 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b. **2**

SECTION – C

The section comprises Short Answer (SA) type questions of **3** marks each.

26. Solve the following Linear Programming problem graphically :

Maximize : $Z = 3x + 3.5y$

subject to constraints : $x + 2y \ge 240$,

$3x + 1.5y \ge 270$,

$1.5x + 2y \le 310$,

$x \ge 0, y \ge 0$. **3**

27. (a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}}\,dx$. **3**

OR

(b) Evaluate $\int_{-a}^{a} f(x)\,dx$, where $f(x) = \frac{9^x}{1+9^x}$. **3**

28. Find $\int e^{\cot^{-1} x}\left(\frac{1-x+x^2}{1+x^2}\right) dx$. **3**

29. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}\,dx$ **3**

30. (a) Find the general solution of the differential equation :

$(xy - x^2)\,dy = y^2\,dx$. **3**

OR

(b) Find the general solution of the differential equation :

$$(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$$ **3**

31. (a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X. **3**

OR

(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not. **3**

SECTION – D

This section comprises Long Answer type (LA) questions of **5** marks each.

32. A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$. **5**

33. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$ **5**

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **5**

34. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. **5**

OR

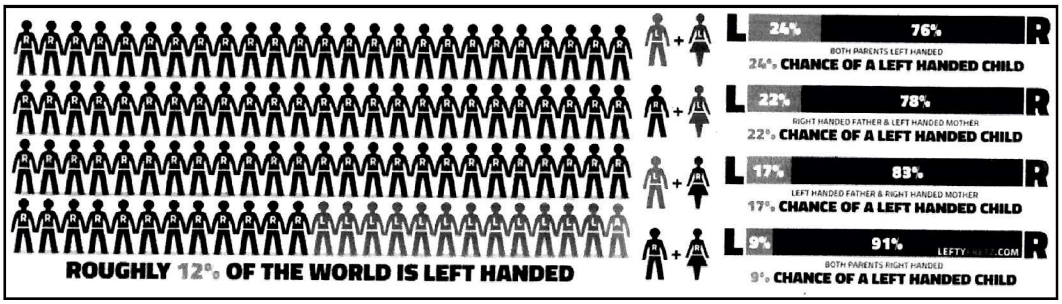
(b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin. **5**

35. Find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x}{5} + \frac{y}{4} = 1$, using integration. **5**

SECTION – E

This section comprises 3 source based case-based/passage based/integrated units of assessment questions of 4 marks each.

36. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$ **1**

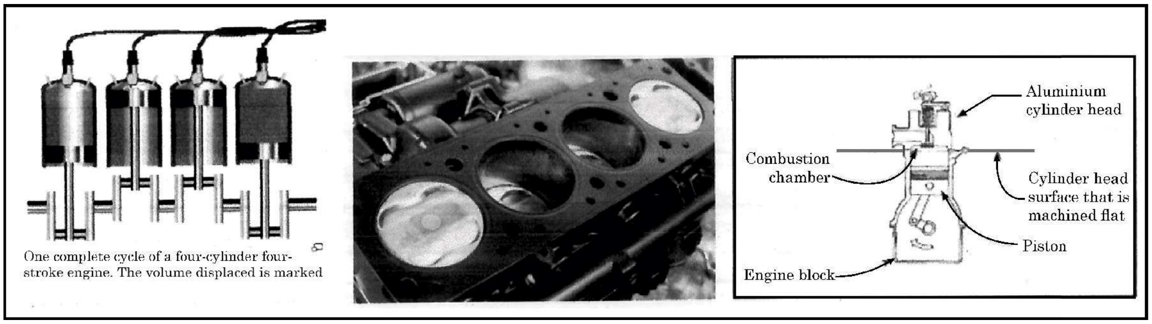
(ii) Find $P(\overline{L}/A)$ **1**

(iii) (a) Find $P(A/L)$ **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

37. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



One complete cycle of a four-cylinder four-stroke engine. The volume displaced is marked

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r. **1**

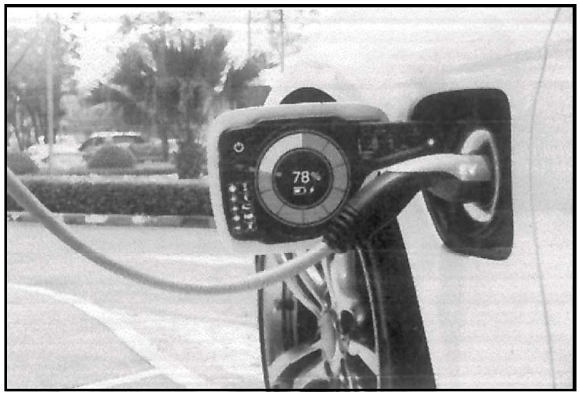
(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h > r. State true or false and justify. **2**

38. The use of electric vehicles will curb air pollution in the long run.



The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**

2023 Annual

Series EF1GH/3

SET~1

प्रश्न–पत्र कोड
Q.P. Code **65/3/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*

Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is :

(a) 10 (b) 6

(c) 8 (d) 0

2. If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

3. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) $AB = O$ (b) $AB = -BA$

(c) $AB = BA$ (d) $BA = O$

4. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{2}$

5. Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

6. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$

(c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^{x}}{\log 2} + C$

7. $\int \frac{2\cos 2x - 1}{1 + 2\sin x} dx$ is equal to :

(a) $x - 2\cos x + C$ (b) $x + 2\cos x + C$

(c) $-x - 2\cos x + C$ (d) $-x + 2\cos x + C$

8. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $\log x - \log y = C$

(c) $xy = C$ (d) $x + y = C$

9. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2} \sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2

(c) 6 (d) not defined

10. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is :

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

11. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

12. In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

13. The value of λ for which the angle between the lines

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and

$\vec{r} = (1+q)\hat{i} + (1+q\lambda)\hat{j} + (1+q)\hat{k}$ is $\frac{\pi}{2}$ is :

(a) -4 (b) 4

(c) 2 (d) -2

14. If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

15. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is :

(a) 1 (b) 2

(c) any real number (d) 0

16. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

17. The number of feasible solutions of the linear programming problem given as

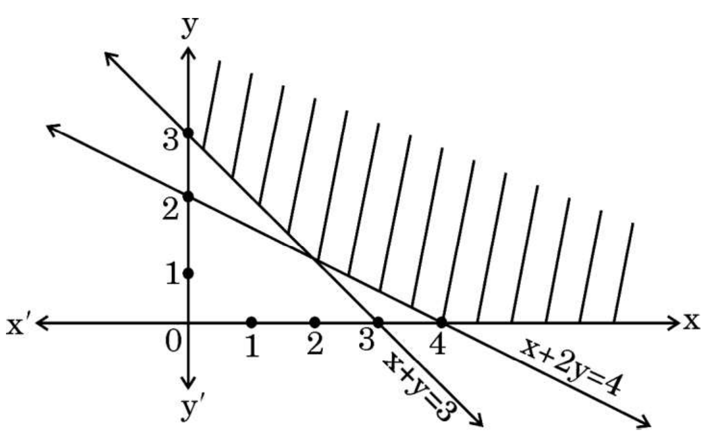
Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

18. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

20. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

22. If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between α and β.

23. If $f(x) = a(\tan x - \cot x)$, where $a > 0$, then find whether f(x) is increasing or decreasing function in its domain.

24. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of $f(x) = \sin^{-1} x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

25. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at x = 1.

OR

(b) If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

OR

(b) Solve the following differential equation :

$$x e^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

27. Evaluate :

$$\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}}\, dx$$

28. Evaluate :

$$\int_1^e \frac{1}{\sqrt{4x^2 - (x \log x)^2}}\, dx$$

29. (a) Find :

$$\int \frac{\cos x}{\sin 3x}\, dx$$

OR

(b) Find :

$$\int x^2 \log (x^2 + 1)\, dx$$

30. Determine graphically the minimum value of the following objective function :

$z = 500x + 400y$

subject to constraints

$x + y \leq 200,$

$x \geq 20,$

$y \geq 4x,$

$y \geq 0.$

31. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Show that a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{5x - 3}{4}$ is both one-one and onto.

33. The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

34. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2\,I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11,\ \ 2x - 7y = -3.$

35. (a) Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

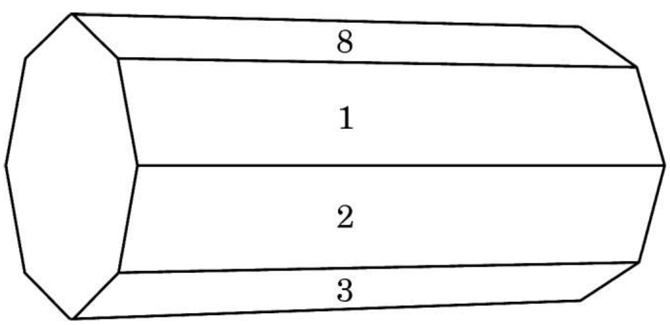
(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. *1*

(ii) Find $P(X > 6)$. *1*

(iii) (a) Find $P(X = 3m)$, where m is a natural number. *2*

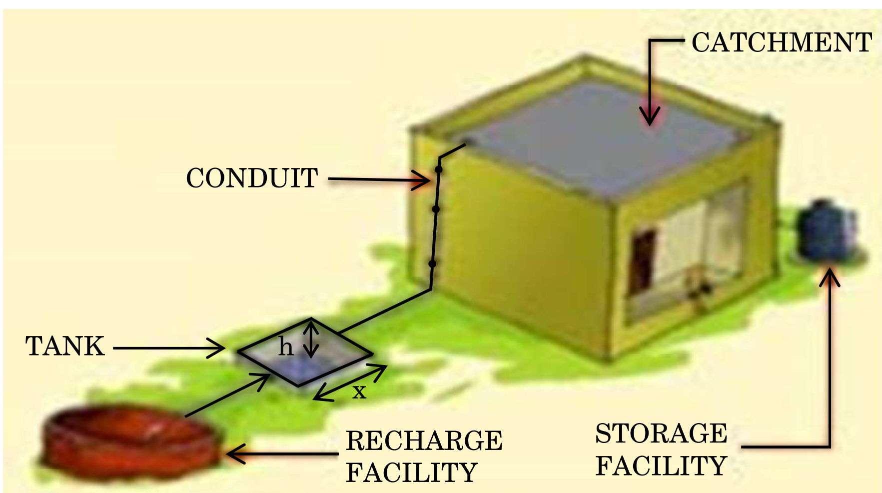
OR

(iii) (b) Find the mean $E(X)$. *2*

Case Study – 2

37. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. *1*

(ii) Find $\frac{dC}{dx}$. *1*

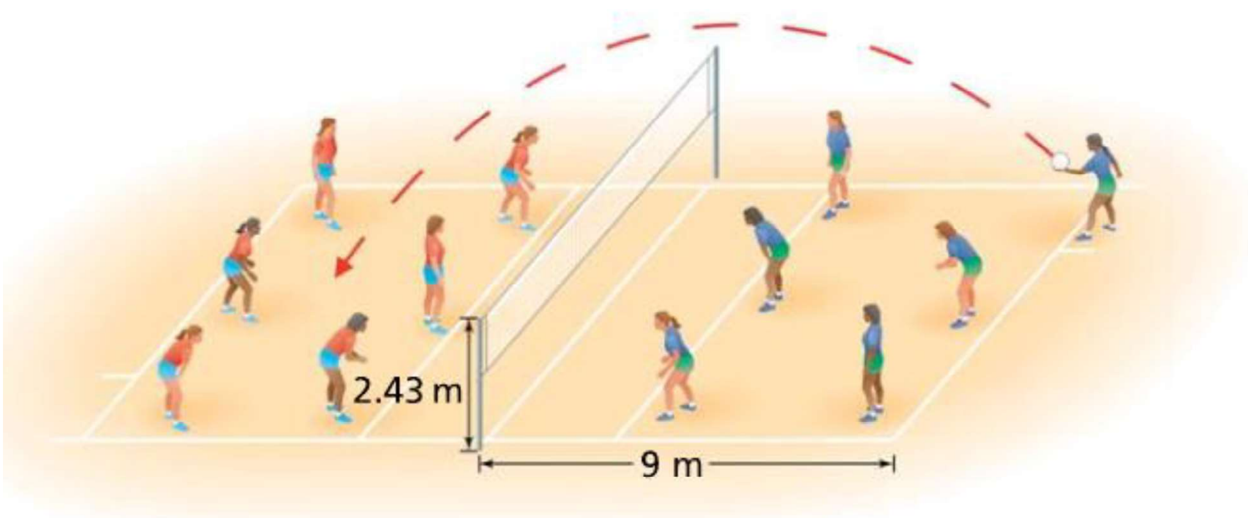
(iii) (a) Find the value of x for which cost C is minimum. *2*

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$. *2*

Case Study – 3

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. *2*

(ii) Find the time at which the height of the ball is maximum. *2*

2023 Annual

Series EF1GH/3 **SET~2**

प्रश्न–पत्र कोड
Q.P. Code **65/3/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS
*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$

(c) $\dfrac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \dfrac{2^x}{\log 2} + C$

2. Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to :

(a) 2023 (b) $\dfrac{1}{2023}$

(c) $(2023)^2$ (d) 1

3. $\int_{0}^{2} \sqrt{4-x^2}\, dx$ equals :

(a) $2 \log 2$ (b) $-2 \log 2$

(c) $\frac{\pi}{2}$ (d) π

4. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is :

(a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $\log x - \log y = C$

(c) $xy = C$ (d) $x + y = C$

5. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}$?

(a) 3 (b) 2

(c) 6 (d) not defined

6. The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are (1, 2, – 1) and (3, 4, 0) respectively, are :

(a) $-2, -2, -1$ (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

(c) $2, 2, 1$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

7. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

8. In Δ ABC, $\overrightarrow{AB}=\hat{i}+\hat{j}+2\hat{k}$ and $\overrightarrow{AC}=3\hat{i}-\hat{j}+4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i}+6\hat{k}$ (b) $2\hat{i}-2\hat{j}+2\hat{k}$

(c) $\hat{i}-\hat{j}+\hat{k}$ (d) $2\hat{i}+3\hat{k}$

9. If the point P(a, b, 0) lies on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$, then (a, b) is :

(a) (1, 2)
(b) $\left(\frac{1}{2}, \frac{2}{3}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$
(d) (0, 0)

10. For any two events A and B, if $P(\overline{A}) = \frac{1}{2}$, $P(\overline{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\overline{A}}{\overline{B}}\right)$ equals :

(a) $\frac{3}{8}$
(b) $\frac{8}{9}$
(c) $\frac{1}{8}$
(d) $\frac{1}{4}$

11. The value of k for which function $f(x)=\begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1
(b) 2
(c) any real number
(d) 0

12. If $y=\frac{\cos x-\sin x}{\cos x+\sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4}-x\right)$
(b) $\sec^2\left(\frac{\pi}{4}-x\right)$
(c) $\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|$
(d) $-\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|$

13. The number of feasible solutions of the linear programming problem given as

Maximize z = 15x + 30y subject to constraints :

$3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is

(a) 1
(b) 2
(c) 3
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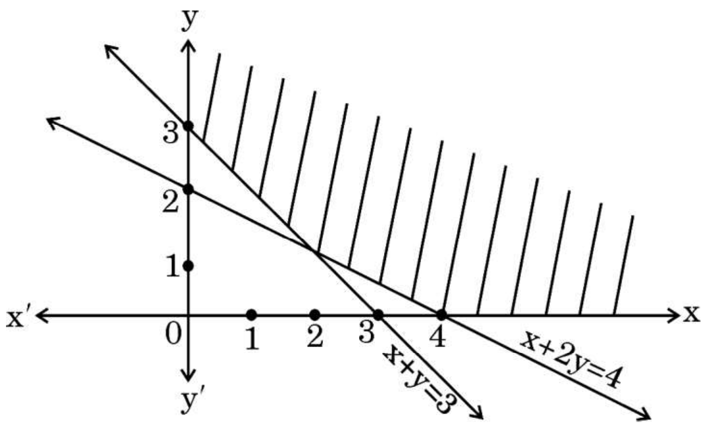
14. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$

(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$

(d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$

15. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $B'A'$ is equal to :

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

16. If A . (adj A) = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

17. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) AB = O (b) AB = – BA

(c) AB = BA (d) BA = O

18. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{2}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

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(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are parallel if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

20. *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Consider the statement "There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}$, $b \neq 0$ is strictly increasing in $\mathbb{R} - \{0\}$."

State True or False. Justify.

22. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$.

23. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

OR

(b) If $x = a\sin 2t$, $y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$.

24. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

25. Find the value of p, so that lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{1-z}{7}$ are perpendicular to each other.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{e^x}{\sqrt{e^{2x} - 4e^x - 5}}\, dx$$

27. (a) Find :

$$\int \frac{\cos x}{\sin 3x}\, dx$$

OR

(b) Find :

$$\int x^2 \log (x^2 + 1)\, dx$$

28. Solve the following linear programming problem graphically :

Maximize $z = 3x + 9y$

subject to the constraints

$$x + y \geq 10,$$
$$x + 3y \leq 60,$$
$$x \leq y,$$
$$x \geq 0,\ y \geq 0.$$

29. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

30. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

OR

(b) Solve the following differential equation :

$$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

31. Evaluate :

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\, dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2\,I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, \; 2x - 7y = -3.$

33. (a) Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

34. Prove that a function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

35. The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

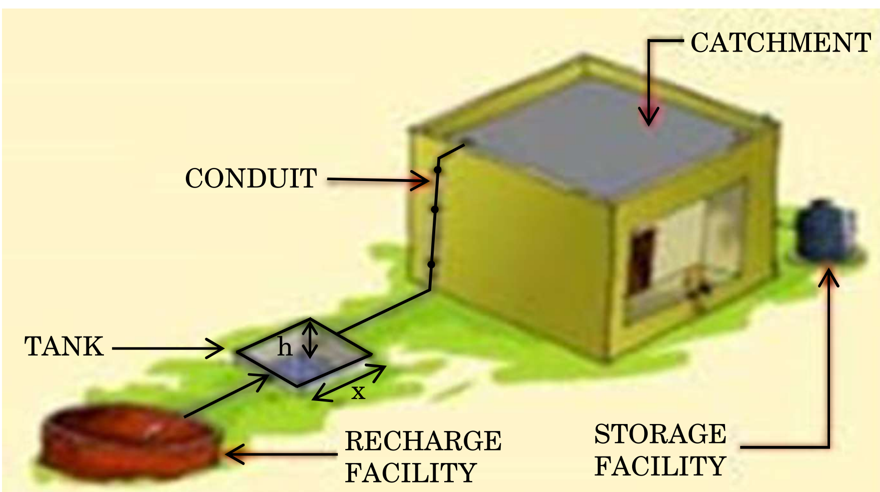
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. *1*

(ii) Find $\frac{dC}{dx}$. *1*

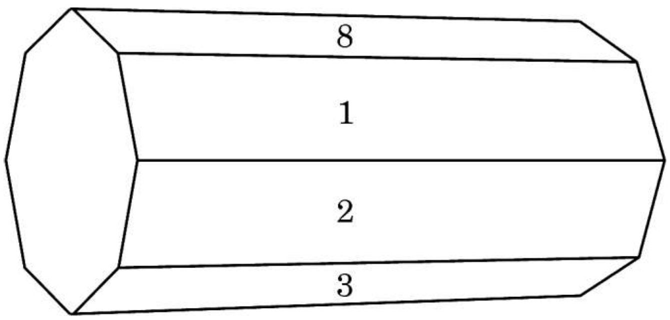
(iii) (a) Find the value of x for which cost C is minimum. *2*

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$. *2*

Case Study – 2

37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. *1*

(ii) Find P(X > 6). *1*

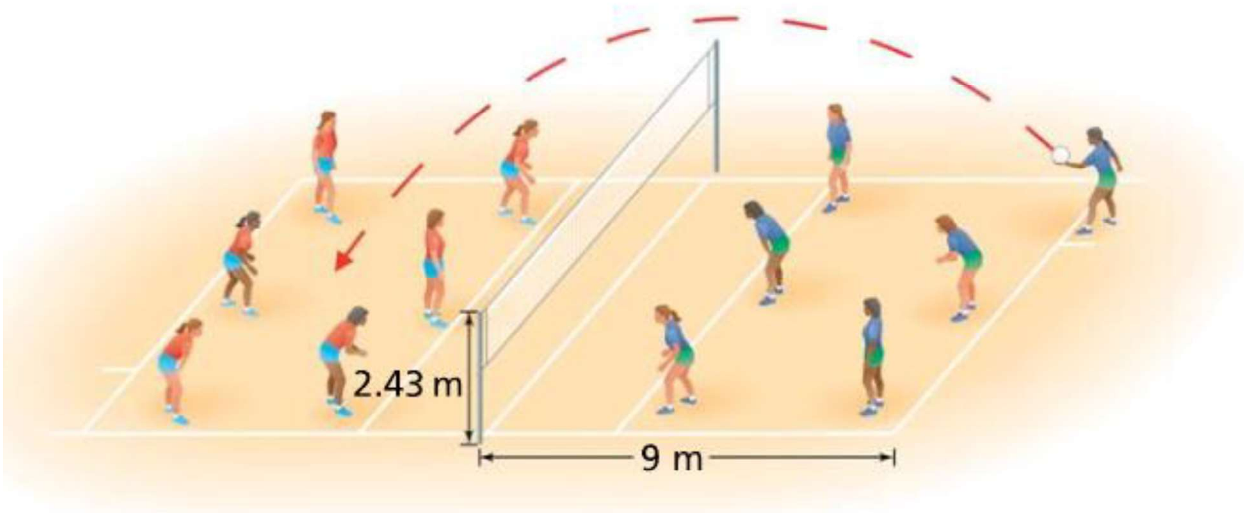
(iii) (a) Find P(X = 3m), where m is a natural number. *2*

OR

(iii) (b) Find the mean E(X). *2*

Case Study – 3

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. *2*

(ii) Find the time at which the height of the ball is maximum. *2*

2023 Annual

Series EF1GH/3 **SET~3**

प्रश्न–पत्र कोड
Q.P. Code **65/3/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to

(a) -1 (b) 1

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

2. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

3. In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

4. The equation of a line passing through point (2, – 1, 0) and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is :

(a) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{2}$ (b) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{2}$

(c) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{-2}$ (d) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$

5. X and Y are independent events such that $P(X \cap \bar{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to :

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

6. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2

(c) any real number (d) 0

7. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

8. The number of feasible solutions of the linear programming problem given as

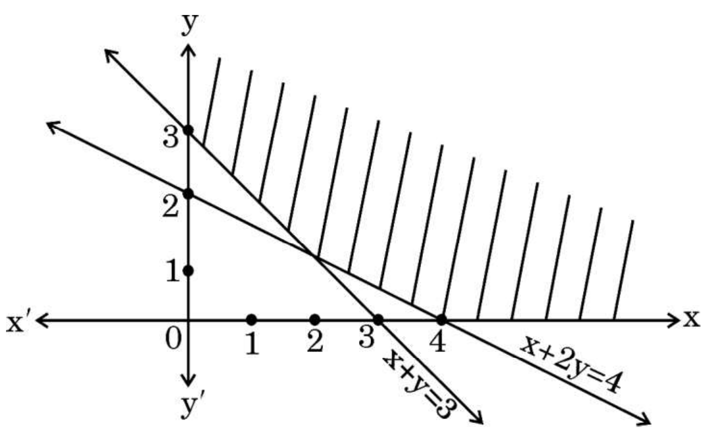
Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0, y \geq 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

9. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x + 2y \geq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$

(b) $x + 2y \leq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$

(c) $x + 2y \geq 4$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$

(d) $x + 2y \geq 4$, $x + y \geq 3$, $x \leq 0$, $y \leq 0$

10. A and B are square matrices of same order. If $(A + B)^2 = A^2 + B^2$, then :

(a) $AB = BA$ (b) $AB = -BA$

(c) $AB = O$ (d) $BA = O$

11. If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

12. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) $AB = O$ (b) $AB = -BA$

(c) $AB = BA$ (d) $BA = O$

13. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where

$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) 0 (d) $\frac{\pi}{2}$

14. Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

15. $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$

(c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^x}{\log 2} + C$

16. $\int e^{-x}\left(\frac{x+1}{x^2}\right) dx$ is equal to :

(a) $\frac{e^{-x}}{x} + C$

(b) $\frac{e^{x}}{x} + C$

(c) $\frac{e^{x}}{x^2} + C$

(d) $-\frac{e^{-x}}{x} + C$

17. The value of $\int_0^{\pi/2} \log \tan x \, dx$ is :

(a) $\frac{\pi}{2}$

(b) 0

(c) $-\frac{\pi}{2}$

(d) 1

18. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2} \sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3

(b) 2

(c) 6

(d) not defined

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

20. *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

OR

(b) If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.

22. If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

23. Find the direction cosines of the line whose Cartesian equations are $5x - 3 = 15y + 7 = 3 - 10z$.

24. Find the points on the curve $6y = x^3 + 2$ at which ordinate is changing 8 times as fast as abscissa.

25. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

OR

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

27. (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

OR

(b) Solve the following differential equation :

$$x e^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

28. Evaluate :

$$\int_0^{\pi/4} \log(1 + \tan x)\, dx$$

29. (a) Find :

$$\int \frac{\cos x}{\sin 3x} dx$$

OR

(b) Find :

$$\int x^2 \log(x^2 + 1)\, dx$$

30. Find :

$$\int_1^4 \frac{1}{\sqrt{2x+1}-\sqrt{2x-1}}\,dx$$

31. Solve the following linear programming problem graphically :

Minimize z = x + 2y

subject to the constraints

$$2x + y \geq 3,$$
$$x + 2y \geq 6,$$
$$x \geq 0,$$
$$y \geq 0.$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.

OR

(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

33. Check whether a function $f : \mathbb{R} \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$ is one-one and onto or not.

34. The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

35. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2\,I = O$.

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, \; 2x - 7y = -3$.

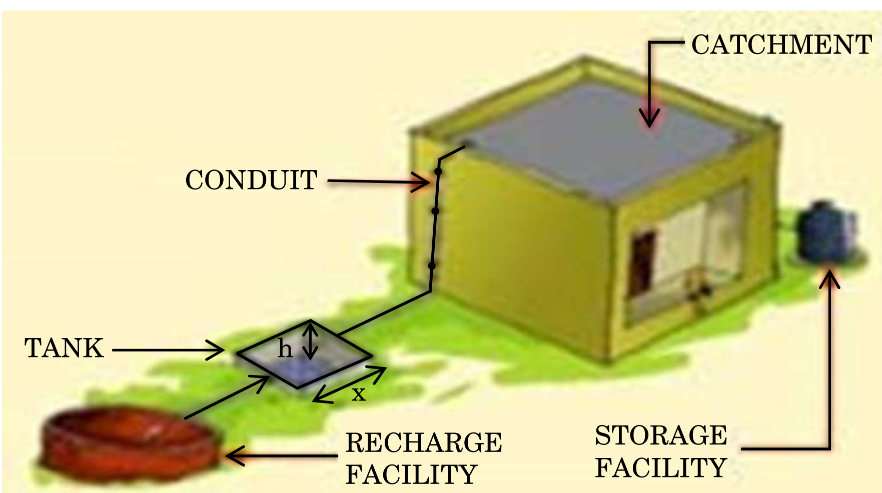
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. *1*

(ii) Find $\frac{dC}{dx}$. *1*

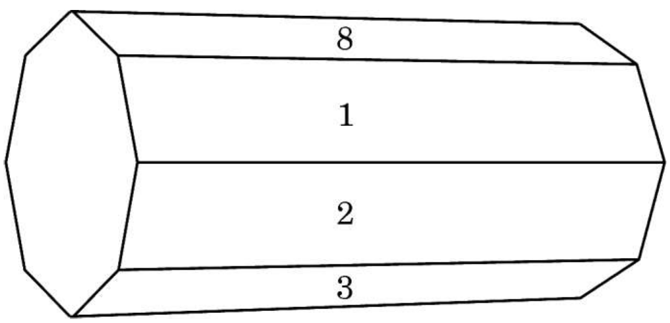
(iii) (a) Find the value of x for which cost C is minimum. *2*

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$. *2*

Case Study – 2

37. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions :

(i) Find the value of p. *1*

(ii) Find $P(X > 6)$. *1*

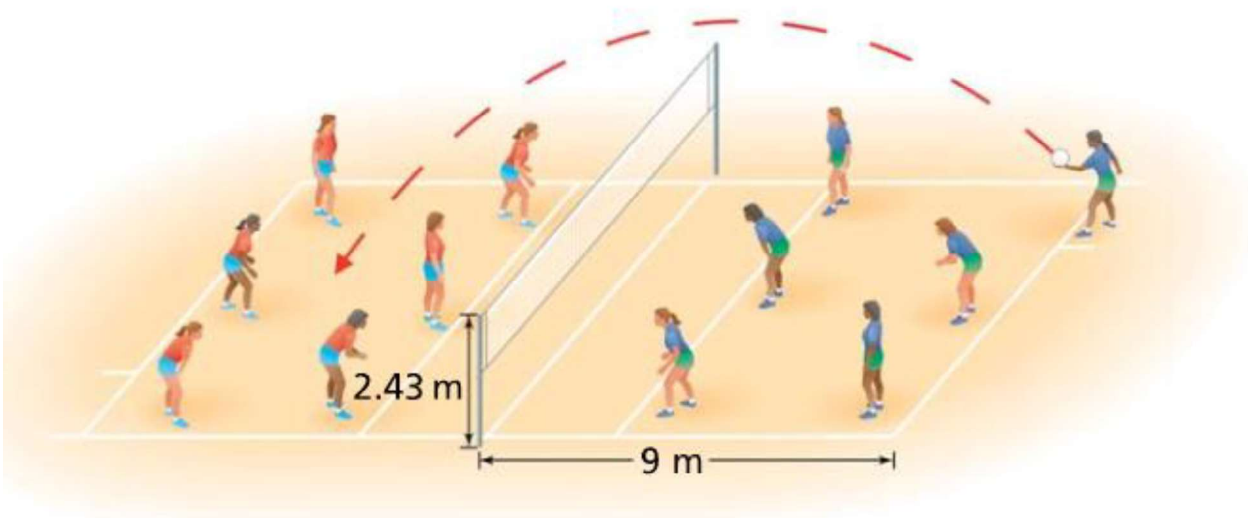
(iii) (a) Find $P(X = 3m)$, where m is a natural number. *2*

OR

(iii) (b) Find the mean E(X). *2*

Case Study – 3

38. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.



Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. *2*

(ii) Find the time at which the height of the ball is maximum. *2*

2023 Annual

Series EF1GH/4 **SET~1**

प्रश्न–पत्र कोड
Q.P. Code **65/4/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*
Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1,\ y = 2$ (b) $x = 2,\ y = 1$

(c) $x = 1,\ y = -1$ (d) $x = 3,\ y = 2$

2. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to :

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

3. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :

(a) I (b) A

(c) 2A (d) 3 I

4. If a matrix $A = [1 \ \ 2 \ \ 3]$, then the matrix AA′ (where A′ is the transpose of A) is :

(a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) $[14]$

5. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is

(a) 0 (b) 1

(c) $x + y + z$ (d) $2(x + y + z)$

6. The function $f(x) = |x|$ is

(a) continuous and differentiable everywhere.

(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at $x = 0$.

(d) continuous everywhere, but differentiable nowhere.

7. If $y = \sin^2 (x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2 (x^3)$

8. $\int e^{5 \log x}\, dx$ is equal to :

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

9. If $\int_0^a 3x^2\, dx = 8$, then the value of 'a' is :

(a) 2 (b) 4

(c) 8 (d) 10

10. The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y} (b) e^{-x}

(c) x (d) $\frac{1}{x}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are :

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

12. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

13. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when :

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

14. Distance of the point (p, q, r) from y-axis is :

(a) q (b) $|q|$

(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

15. The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

16. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

(a) $(-2, 4)$ (b) $(3, 2)$

(c) $(-5, 6)$ (d) $(4, 2)$

17. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then :

(a) $0 < a < 1$ (b) $a > 2$

(c) $a > 0$ (d) $a = \pm\sqrt{3}$

18. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$

(c) $\frac{3}{20}$ (d) $\frac{4}{5}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

20. *Assertion (A) :* The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular, when $\vec{b}_1 \cdot \vec{b}_2 = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by $\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

22. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

23. Find the maximum and minimum values of the function given by $f(x) = 5 + \sin 2x$.

24. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} \, dx$$

27. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1 + 5^x} \, dx$$

28. (a) Find :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} \, dx$$

OR

(b) Evaluate :

$$\int_{0}^{\pi/2} \sqrt{\sin x} \cos^5 x \, dx$$

29. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x + y}{x}, \; y(1) = 0.$$

OR

(b) Find the general solution of the differential equation

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.$$

30. Solve the following linear programming problem graphically :

Minimise : $z = -3x + 4y$

subject to the constraints

$$x + 2y \leq 8,$$
$$3x + 2y \leq 12,$$
$$x, y \geq 0.$$

31. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3.$

33. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

34. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by (a, b) R (c, d), if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

35. (a) Show that the following lines do not intersect each other :

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5};\ \frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$$

OR

(b) Find the angle between the lines

$2x = 3y = -z$ and $6x = -y = -4z$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find $f'(2)$ and $f'(-1)$. *2*

Case Study – 2

37. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

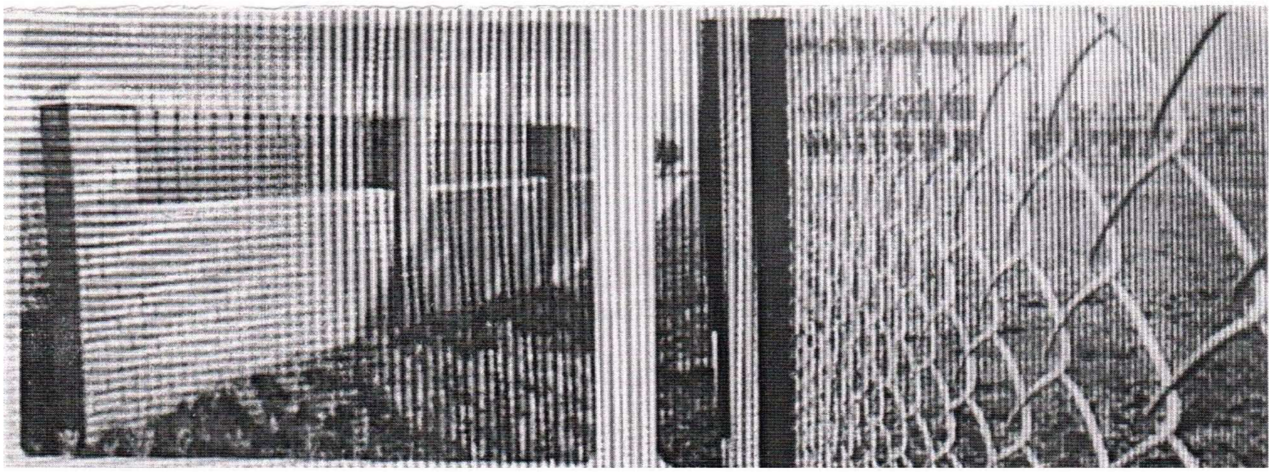
(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*

2023 Annual

Series EF1GH/4 **SET~2**

प्रश्न–पत्र कोड **Q.P. Code 65/4/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

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(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then the value of $(2x + y - z)$ is :

(a) 1 (b) 2

(c) 3 (d) 5

2. If a matrix $A = [1 \;\; 2 \;\; 3]$, then the matrix AA' (where A' is the transpose of A) is :

(a) 14

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

(d) $[14]$

3. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1,\ y = 2$ (b) $x = 2,\ y = 1$

(c) $x = 1,\ y = -1$ (d) $x = 3,\ y = 2$

4. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :

(a) I (b) A

(c) 2A (d) 3 I

5. The value of the determinant $\begin{vmatrix}2 & 7 & 1\\1 & 1 & 1\\10 & 8 & 1\end{vmatrix}$ is :

(a) 47 (b) – 79

(c) 49 (d) – 51

6. The function $f(x) = |x|$ is

(a) continuous and differentiable everywhere.

(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at $x = 0$.

(d) continuous everywhere, but differentiable nowhere.

7. If $y = \log(\sin e^x)$, then $\dfrac{dy}{dx}$ is :

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$

(c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

8. $\int e^{5 \log x}\, dx$ is equal to :

(a) $\dfrac{x^5}{5} + C$ (b) $\dfrac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

9. $\int_0^4 (e^{2x} + x)\, dx$ is equal to :

(a) $\frac{15 + e^8}{2}$ (b) $\frac{16 - e^8}{2}$

(c) $\frac{e^8 - 15}{2}$ (d) $\frac{-e^8 - 15}{2}$

10. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

11. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when :

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

12. The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y} (b) e^{-x}

(c) x (d) $\frac{1}{x}$

13. The number of solutions of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$, is :

(a) zero (b) one

(c) two (d) infinite

14. Distance of the point (p, q, r) from y-axis is :

(a) q (b) $|q|$

(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

15. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then :

(a) $0 < a < 1$ (b) $a > 2$

(c) $a > 0$ (d) $a = \pm\sqrt{3}$

16. For two events A and B, if P(A) = 0·4, P(B) = 0·8 and P(B/A) = 0·6, then $P(A \cup B)$ is :

(a) 0·24 (b) 0·3

(c) 0·48 (d) 0·96

17. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

(a) (– 2, 4) (b) (3, 2)

(c) (– 5, 6) (d) (4, 2)

18. The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

20. *Assertion (A) :* The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular, when $\vec{b}_1 \cdot \vec{b}_2 = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by $\cos\theta = \dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Find the interval in which the function $f(x) = 2x^3 - 3x$ is strictly increasing.

22. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

23. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

24. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

25. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector along the vector $\vec{a} \times \vec{b}$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^2}{x^2 + 6x + 12} \, dx$$

27. Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X.

28. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, \quad y(1) = 0.$$

OR

(b) Find the general solution of the differential equation

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.$$

29. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1+5^x}\, dx$$

30. (a) Find :

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\, dx$$

OR

(b) Evaluate :

$$\int_{0}^{\pi/2} \sqrt{\sin x}\, \cos^5 x\, dx$$

31. Solve the following linear programming problem graphically :

Maximise $z = -3x - 5y$

subject to the constraints

$$-2x + y \le 4,$$

$$x + y \ge 3,$$

$$x - 2y \le 2,$$

$$x \ge 0,\ y \ge 0.$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y-axis, but lying in the 1st quadrant.

33. (a) Show that the following lines do not intersect each other :

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}; \quad \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$$

OR

(b) Find the angle between the lines

$$2x = 3y = -z \text{ and } 6x = -y = -4z.$$

34. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

35. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1;\ \ 2y - 3z = 1;\ \ 3x - 2y + 4z = 3.$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Case Study – 2

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim\limits_{h \to 0} \dfrac{f(a-h) - f(a)}{-h}$

- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

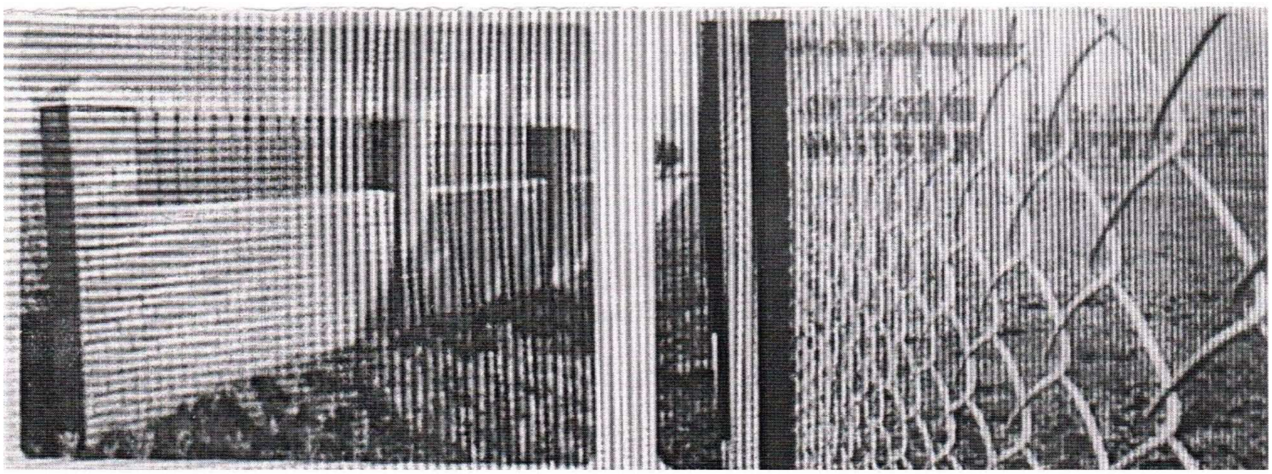
(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find f′(2) and f′(– 1). *2*

Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*

2023 Annual

Series EF1GH/4 **SET~3**

प्रश्न–पत्र कोड
Q.P. Code **65/4/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS
*

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*
Time allowed : 3 hours — *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***
*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If A is a 3×4 matrix and B is a matrix such that A′B and AB′ are both defined, then the order of the matrix B is :

(a) 3×4 (b) 3×3

(c) 4×4 (d) 4×3

2. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq units, then k is

(a) 12 (b) -2

(c) $-12, -2$ (d) $12, -2$

3. If $f(x) = 2|x| + 3|\sin x| + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is :

(a) 6 (b) 5

(c) 3 (d) 2

4. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1,\ y = 2$ (b) $x = 2,\ y = 1$

(c) $x = 1,\ y = -1$ (d) $x = 3,\ y = 2$

5. If a matrix $A = [1\ \ 2\ \ 3]$, then the matrix AA' (where A' is the transpose of A) is :

(a) 14 (b) $\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$

(c) $\begin{bmatrix}1&2&3\\2&3&1\\3&1&2\end{bmatrix}$ (d) $[14]$

6. The product $\begin{bmatrix}a&b\\-b&a\end{bmatrix}\begin{bmatrix}a&-b\\b&a\end{bmatrix}$ is equal to :

(a) $\begin{bmatrix}a^2+b^2&0\\0&a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2&0\\(a+b)^2&0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2&0\\a^2+b^2&0\end{bmatrix}$ (d) $\begin{bmatrix}a&0\\0&b\end{bmatrix}$

7. Distance of the point (p, q, r) from y-axis is :

(a) q (b) $|q|$

(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

8. The solution set of the inequation $3x + 5y < 7$ is :

(a) whole xy-plane except the points lying on the line $3x + 5y = 7$.

(b) whole xy-plane along with the points lying on the line $3x + 5y = 7$.

(c) open half plane containing the origin except the points of line $3x + 5y = 7$.

(d) open half plane not containing the origin.

9. If $\int_0^a 3x^2 \, dx = 8$, then the value of 'a' is :

(a) 2 (b) 4

(c) 8 (d) 10

10. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$

(c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are :

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

12. $\int e^{5 \log x} \, dx$ is equal to :

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

13. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

14. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

(a) $(-2, 4)$ (b) $(3, 2)$

(c) $(-5, 6)$ (d) $(4, 2)$

15. If $y = \sin^2 (x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2 (x^3)$

16. The point $(x, y, 0)$ on the xy-plane divides the line segment joining the points $(1, 2, 3)$ and $(3, 2, 1)$ in the ratio :

(a) 1 : 2 internally (b) 2 : 1 internally

(c) 3 : 1 internally (d) 3 : 1 externally

17. The events E and F are independent. If $P(E) = 0\cdot3$ and $P(E \cup F) = 0\cdot5$, then $P(E/F) - P(F/E)$ equals :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{35}$ (d) $\frac{1}{70}$

18. The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y} (b) e^{-x}

(c) x (d) $\frac{1}{x}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by $\cos \theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

20. *Assertion (A) :* All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If $xy = e^{x-y}$, then show that $\dfrac{dy}{dx} = \dfrac{y(x-1)}{x(y+1)}$.

22. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

23. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

24. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{2}{(1-x)(1+x^2)}\,dx$$

27. (a) Evaluate :

$$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\,dx$$

OR

(b) Evaluate :

$$\int_{1}^{3} \{|(x-1)| + |(x-2)|\}\,dx$$

28. Solve the following linear programming problem graphically :

Maximise z = 5x + 3y

subject to the constraints

$3x + 5y \leq 15,$

$5x + 2y \leq 10,$

$x, y \geq 0.$

29. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

30. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, \; y(1) = 0.$$

OR

(b) Find the general solution of the differential equation

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.$$

31. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1 + 5^x} \, dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the image of the point $(2, -1, 5)$ in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

OR

(b) Vertices B and C of Δ ABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of Δ ABC given that point A has coordinates $(1, -1, 2)$ and the line segment BC has length of 5 units.

33. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1; \; 2y - 3z = 1; \; 3x - 2y + 4z = 3.$

34. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

35. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by (a, b) R (c, d), if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Case Study – 2

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

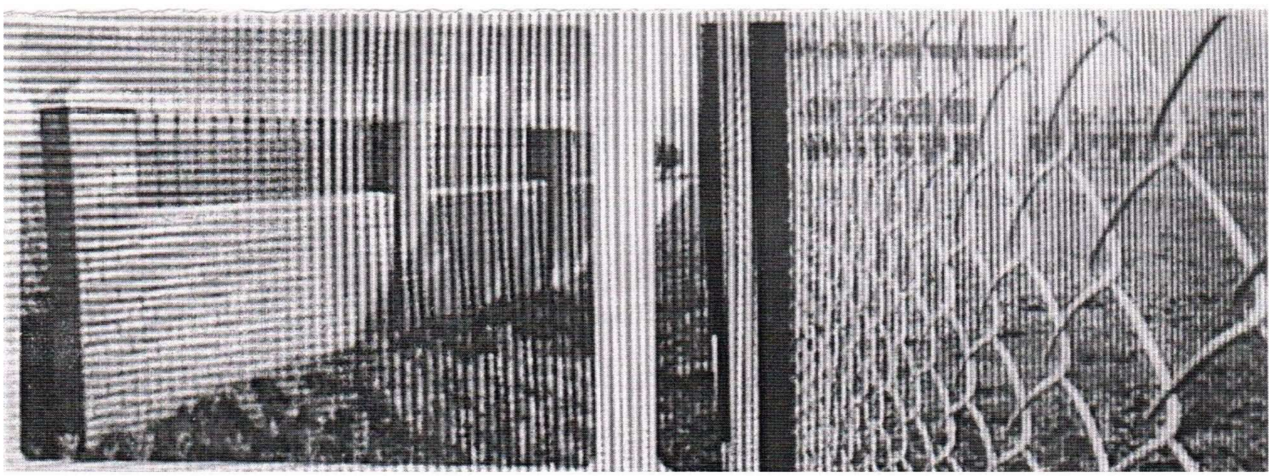
(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find the $f'(2)$ and $f'(-1)$. *2*

Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*

2023 Annual

Series EF1GH/5 **SET~1**

प्रश्न-पत्र कोड
Q.P. Code ***65/5/1***

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

*Time allowed : **3** hours* *Maximum Marks : **80***

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are compulsory.*

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B, C, D*** *and* ***E****.*

(iii) *In* **Section A** *– Question Number* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQ) type and Question Number* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* **Section B** *– Question Number* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions of* ***2*** *marks each.*

(v) *In* **Section C** *– Question Number* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* **Section D** *– Question Number* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* **Section E** *– Question Number* ***36*** *to* ***38*** *are case study based questions carrying 4 marks each where* ***2 VSA*** *type questions are of* ***1*** *mark each and* ***1*** *SA type question is of* ***2*** *marks. Internal choice is provided in* ***2*** *marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section –* ***B****,* ***3*** *questions in Section –* ***C****, 2 questions in Section –* ***D*** *and* ***2*** *questions in Section –* ***E****.*

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries **1** mark.

Select the correct option out of the four given options :

1. Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

2. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

3. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

(a) A (b) A + I

(c) I – A (d) A – I

4. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals

(a) ± 1 (b) –1

(c) 1 (d) 2

5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1 (b) 2

(c) 3 (d) 4

6. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

7. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$

(c) $x = -2$ (d) $x = 4$

8. If $x = A \cos 4t + B \sin 4t$, then $\dfrac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$

(c) $16x$ (d) $-16x$

9. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $[-1, 1]$

10. $\displaystyle\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

11. $\displaystyle\int_{-1}^{1} \frac{|x-2|}{x-2}\, dx$, $x \neq 2$ is equal to

(a) 1 (b) -1

(c) 2 (d) -2

12. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3

(c) 5 (d) 0

13. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1 (b) 5

(c) 7 (d) 12

15. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30°

(c) 45° (d) 90°

17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

18. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

Assertion – Reason Based Questions

In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

20. **Assertion (A) :** $\int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$

Reason (R) : $\int_{a}^{b} f(x)\, dx = \int_{a}^{b} f(a+b-x)\, dx$

SECTION – B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Write the domain and range (principle value branch) of the following functions :

$f(x) = \tan^{-1} x$

22. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(b) Find the value(s) of 'λ', if the function

$$f(x) = \begin{cases} \dfrac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

23. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

24. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

25. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

SECTION – C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

27. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

28. (a) Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} \, dx$

OR

(b) Find : $\int \frac{x^4}{(x-1)(x^2+1)} \, dx$

29. Find the area of the following region using integration :

$\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$

30. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

OR

(b) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$.

31. Find the distance between the lines :

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})\,;$$

$$\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(4\hat{i}+6\hat{j}+12\hat{k})$$

SECTION – D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

33. Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x) \, dx$

34. Solve the following Linear Programming Problem graphically :

Maximize : $P = 70x + 40y$

subject to : $3x + 2y \leq 9,$

$3x + y \leq 9,$

$x \geq 0, y \geq 0$

35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

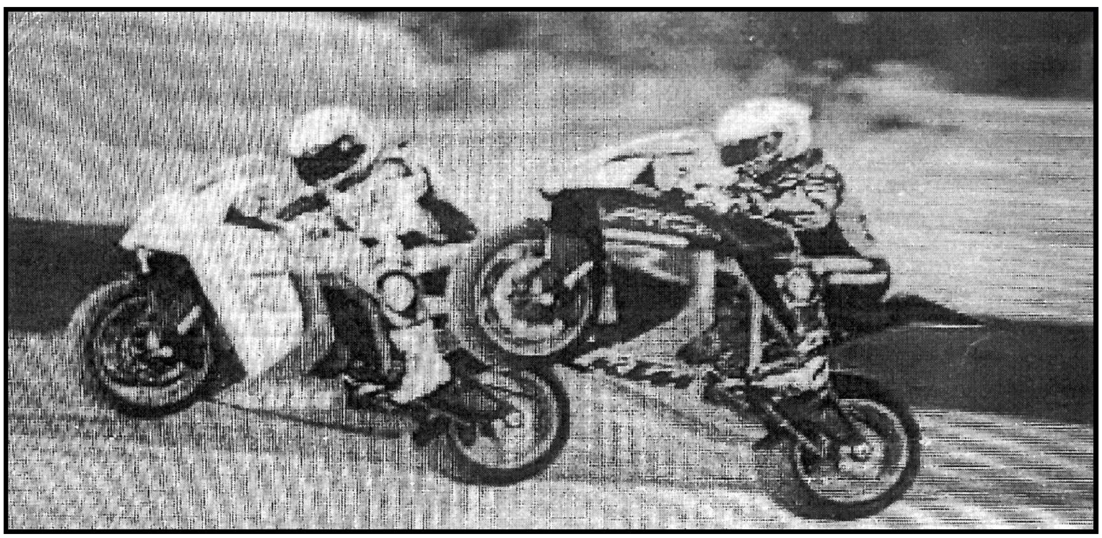
SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub – parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

2023 Annual

Series EF1GH/5 **SET~2**

प्रश्न-पत्र कोड
Q.P. Code **65/5/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This question paper contains **38** questions. **All** questions are compulsory.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions carrying 4 marks each where **2 VSA** type questions are of **1** mark each and **1** SA type question is of **2** marks. Internal choice is provided in **2** marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculators is **NOT** allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries **1** mark.

Select the correct option out of the four given options :

1. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

3. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals

(a) ± 1 (b) -1

(c) 1 (d) 2

4. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then A^2 is

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(a) 10 (b) 8

(c) 7 (d) -7

6. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$

(c) $x = -2$ (d) $x = 4$

7. The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

8. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $[-1, 1]$

9. The function $f(x) = x \mid x \mid, x \in R$ is differentiable

(a) only at $x = 0$ (b) only at $x = 1$

(c) in R (d) in $R - \{0\}$

10. $\int \frac{\sec x}{\sec x - \tan x} \, dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

11. The value of $\int_0^{\frac{\pi}{4}} (\sin 2x) \, dx$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

12. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3

(c) 5 (d) 0

13. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

15. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30°

(c) 45° (d) 90°

16. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

18. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'/A)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{3}{4}$ (d) 1

Assertion – Reason Based Questions

In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx = 3$

Reason (R) : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

20. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

SECTION – B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$.

22. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

23. (a) If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(b) Find the value(s) of 'λ', if the function

$$f(x) = \begin{cases} \dfrac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

24. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

25. Find the angle between the following two lines :

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k});$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

SECTION – C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. Using determinants, find the area of ΔPQR with vertices P(3, 1), Q(9, 3) and R(5, 7). Also, find the equation of line PQ using determinants.

27. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

28. (a) Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}\, dx$

OR

(b) Find : $\int e^{x^2}(x^5 + 2x^3)\, dx$

29. Find the area of the minor segment of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$, using integration.

30. Find the distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

31. (a) Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

OR

(b) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

SECTION – D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$

33. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

34. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

35. Solve the following Linear Programming Problem graphically :

Maximize : $P = 70x + 40y$

subject to : $3x + 2y \leq 9,$

$3x + y \leq 9,$

$x \geq 0,\ y \geq 0$

SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub – parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find $|A|$.

(III) Find A^{-1}.

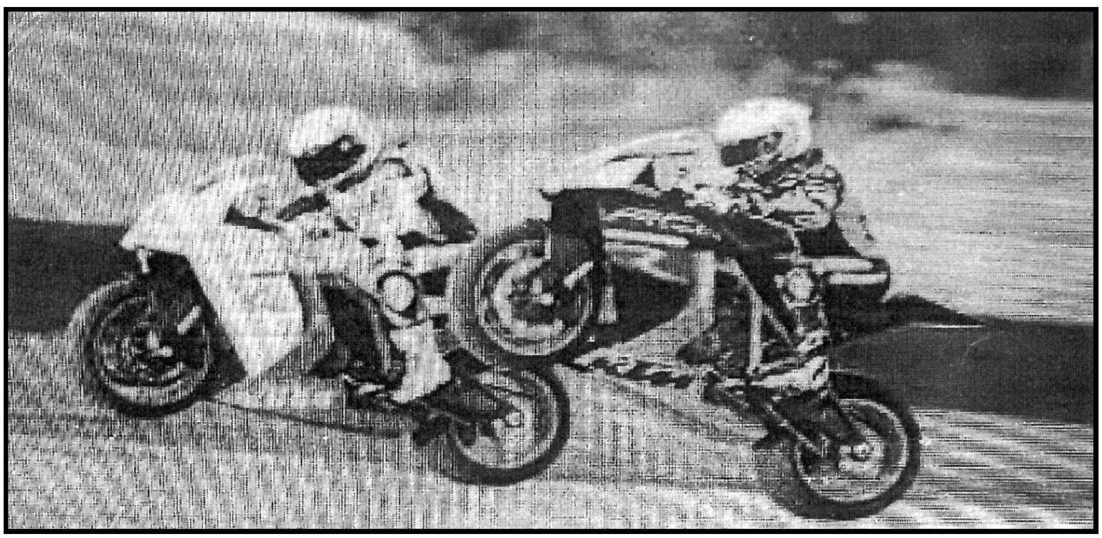
OR

(III) Determine $P = A^2 - 5A$.

Case Study-II

37. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let R : B $\rightarrow$ B be defined by R = $\{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function f : B $\rightarrow$ G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\, dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

2023 Annual

Series EF1GH/5

SET~3

प्रश्न-पत्र कोड
Q.P. Code ***65/5/3***

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***

*Time allowed : **3** hours* | *Maximum Marks : **80***

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are compulsory.*

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B, C, D*** *and* ***E.***

(iii) *In* **Section A** *– Question Number* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQ) type and Question Number* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* **Section B** *– Question Number* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions of* ***2*** *marks each.*

(v) *In* **Section C** *– Question Number* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* **Section D** *– Question Number* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* **Section E** *– Question Number* ***36*** *to* ***38*** *are case study based questions carrying 4 marks each where* ***2 VSA*** *type questions are of* ***1*** *mark each and* ***1*** *SA type question is of* ***2*** *marks. Internal choice is provided in* ***2*** *marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in* ***2*** *questions in Section –* ***B, 3*** *questions in Section –* ***C,*** *2 questions in Section –* ***D*** *and* ***2*** *questions in Section –* ***E.***

(ix) *Use of calculators is* ***NOT*** *allowed.*

SECTION – A

(Multiple Choice Questions)

Each question carries **1** mark.

Select the correct option out of the four given options :

1. Let R be a relation in the set N given by

   $R = \{(a, b) : a = b - 2, b > 6\}$.

   Then

   (a) $(8, 7) \in R$ (b) $(6, 8) \in R$

   (c) $(3, 8) \in R$ (d) $(2, 4) \in R$

2. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, where A^T is the transpose of the matrix A, then

   (a) $x = 0, y = 5$ (b) $x = y$

   (c) $x + y = 5$ (d) $x = 5, y = 0$

3. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

   (a) 1 (b) $\frac{1}{2}$

   (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

4. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

   (a) A (b) A + I

   (c) I – A (d) A – I

5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

   (a) 1 (b) 2

   (c) 3 (d) 4

6. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is

(a) 1 (b) –1

(c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

7. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$

(c) $16x$ (d) $-16x$

8. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$

(c) $x = -2$ (d) $x = 4$

9. The function $f(x) = x^3 + 3x$ is increasing in interval

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $\mathbb{R}$ (d) $(0, 1)$

10. $\int_{-1}^{1} \frac{|x-2|}{x-2}\, dx$, $x \neq 2$ is equal to

(a) 1 (b) –1

(c) 2 (d) –2

11. $\int \frac{\sec x}{\sec x - \tan x}\, dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$

(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

12. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are :

(a) $1, \frac{2}{3}$ (b) 3, 1

(c) 3, 3 (d) 1, 2

13. If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$, then $\vec{a}$ is

(a) $\hat{k}$ (b) $\hat{i}$

(c) $\hat{j}$ (d) $\hat{i} + \hat{j} + \hat{k}$

14. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

15. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30°

(c) 45° (d) 90°

17. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$

(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

18. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1

(b) 5

(c) 7

(d) 12

Assertion – Reason Based Questions

In the following questions **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false.

(d) (A) is false, but (R) is true.

19. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$

Reason (R) : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

20. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

SECTION – B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Find the value of k for which the function f given as

$$f(x) = \begin{cases} \dfrac{1 - \cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

OR

(b) If $x = a \cos t$ and $y = b \sin t$, then find $\dfrac{d^2y}{dx^2}$.

22. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\dfrac{1}{2}\right)\right] + \tan^{-1} 1$.

23. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

24. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

25. (a) If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \dfrac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

SECTION – C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

27. Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis.

28. (a) Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$.

OR

(b) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

29. Find the distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) ;$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

30. (a) Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

OR

(b) If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.

31. (a) Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

OR

(b) Find : $\int \frac{x^4}{(x-1)(x^2+1)}\, dx$

SECTION – D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. Solve the following Linear Programming Problem graphically :

Minimise : $Z = 60x + 80y$

subject to constraints :

$3x + 4y \geq 8$

$5x + 2y \geq 11$

$x, y \geq 0$

33. (a) The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

(b) Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

34. Evaluate : $\int_{0}^{\frac{\pi}{2}} \sin 2x \tan^{-1} (\sin x) \, dx$

35. (a) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

OR

(b) A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

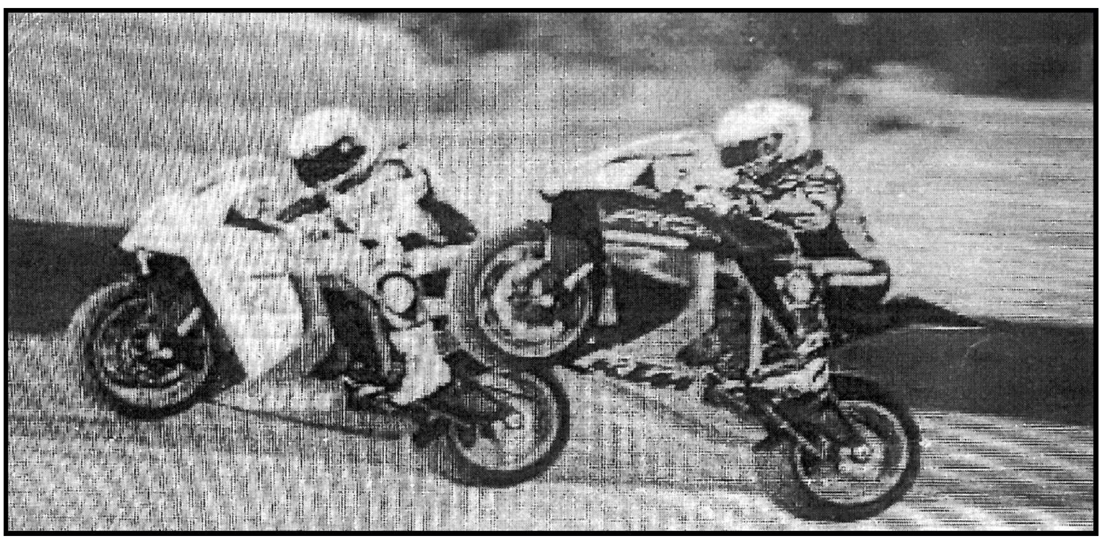
SECTION – E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub – parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub – parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

Based on the above, answer the following questions :

(I) Show that $(x^2 - y^2)\, dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

2022 Compt.

Series A6BAB/C

Set No. 1

प्रश्न-पत्र कोड
Q.P. Code **65/6/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Question numbers ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. Evaluate : *2*

$$\int_0^{\pi/2} \frac{1}{1+\cot^{5/2} x}\,dx$$

2. If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$. *2*

3. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7. *2*

4. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. *2*

5. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. *2*

6. (a) Find the general solution of the differential equation

$$x \cos y \, dy = (x \log x + 1) e^x \, dx.$$ *2*

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order and the degree of the differential equation

$$x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0.$$ *2*

SECTION B

*Question numbers **7 to 10** carry **3** marks each.*

7. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \leq 9, \ x + y \geq 3\}$, using integration. *3*

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis. *3*

8. Find : *3*

$$\int \frac{\sin x}{\sin(x - 2a)} \, dx$$

9. Find the equation of the plane passing through three points whose position vectors are $-\hat{j}$, $3\hat{i} + 3\hat{j}$ and $\hat{i} + \hat{j} + \hat{k}$. *3*

10. (a) Find the distance between the following parallel lines : *3*

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$
$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points (– 1, 1, – 8) and (5, – 2, 10) crosses the ZX-plane. *3*

SECTION C

*Question numbers **11** to **14** carry **4** marks each.*

11. Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3). *4*

12. (a) Find : *4*

$$\int \cos x \cdot \tan^{-1} (\sin x)\, dx$$

OR

(b) Find : *4*

$$\int \frac{e^x}{(e^x + 1)(e^x + 3)} dx$$

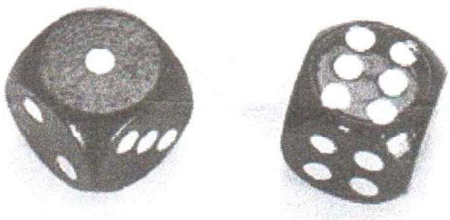
13. Find the particular solution of the differential equation

$$x \frac{dy}{dx} + 2y = x^2 \log x, \text{ given } y(1) = 1.$$ *4*

Case-Study Based Question

14. A biased die is tossed and respective probabilities for various faces to turn up are the following :

Face	1	2	3	4	5	6
Probability	0·1	0·24	0·19	0·18	0·15	K



Based on the above information, answer the following questions :

(a) What is the value of K ? *2*

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4 ? *2*

2022 Compt.

Series A6BAB/C **Set No. 2**

प्रश्न-पत्र कोड
Q.P. Code **65/6/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे — *अधिकतम अंक : 40*

Time allowed : 2 hours — *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Question numbers **1** to **6** carry **2** marks each.*

1. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7. *2*

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. *2*

3. (a) Find the general solution of the differential equation

$$x \cos y \, dy = (x \log x + 1) \, e^x \, dx.$$ *2*

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order and the degree of the differential equation

$$x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0.$$ *2*

4. Evaluate : *2*

$$\int_0^5 x \cdot \sqrt{5 - x} \, dx$$

5. If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$. *2*

6. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. *2*

SECTION B

*Question numbers **7** to **10** carry **3** marks each.*

7. (a) Find the distance between the following parallel lines : *3*

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$
$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX-plane. *3*

8. Find the coordinates of the foot of the perpendicular drawn from the point $(1, 3, 4)$ to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$. *3*

9. Find : *3*

$$\int \sin^{-1} x \, dx$$

10. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \leq 9, \ x + y \geq 3\}$, using integration. *3*

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis. *3*

SECTION C

*Question numbers **11** to **14** carry **4** marks each.*

11. Find the particular solution of the differential equation

$$(1 + \sin x)\frac{dy}{dx} = -x - y\cos x, \text{ given } y(0) = 1. \quad 4$$

12. Find the equation of the plane passing through the intersection of the planes $\vec{r}\cdot(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r}\cdot(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3). *4*

13. (a) Find : *4*

$$\int \cos x \cdot \tan^{-1}(\sin x)\, dx$$

OR

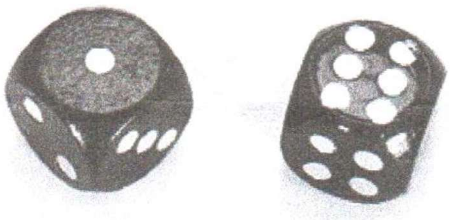
(b) Find : *4*

$$\int \frac{e^x}{(e^x + 1)(e^x + 3)}\, dx$$

Case-Study Based Question

14. A biased die is tossed and respective probabilities for various faces to turn up are the following :

Face	1	2	3	4	5	6
Probability	0·1	0·24	0·19	0·18	0·15	K



Based on the above information, answer the following questions :

(a) What is the value of K ? *2*

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4 ? *2*

2022 Compt.

Series A6BAB/C

Set No. 3

प्रश्न-पत्र कोड
Q.P. Code **65/6/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
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- *Please check that this question paper contains **14** questions.*
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गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Question numbers **1** to **6** carry **2** marks each.*

1. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. *2*

2. (a) Find the general solution of the differential equation

$$x \cos y \, dy = (x \log x + 1) e^x \, dx.$$ *2*

OR

(b) Find the value of $(2a - 3b)$, if a and b represent respectively the order and the degree of the differential equation

$$x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0.$$ *2*

3. Evaluate : *2*

$$\int_1^2 \log\left(\frac{3}{x} - 1\right) dx$$

4. If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$. *2*

5. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. *2*

6. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7. *2*

SECTION B

*Question numbers **7** to **10** carry **3** marks each.*

7. Find the distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x + y + z = 17$. *3*

8. (a) Find the distance between the following parallel lines : *3*

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX-plane. *3*

9. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \le 9,\ x + y \ge 3\}$, using integration. *3*

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis. *3*

10. Find : *3*

$$\int \sin 2x \sin 3x \, dx$$

SECTION C

*Question numbers **11** to **14** carry **4** marks each.*

11. (a) Find : *4*

$$\int \cos x \,.\, \tan^{-1} (\sin x)\, dx$$

OR

(b) Find : *4*

$$\int \frac{e^x}{(e^x + 1)(e^x + 3)} dx$$

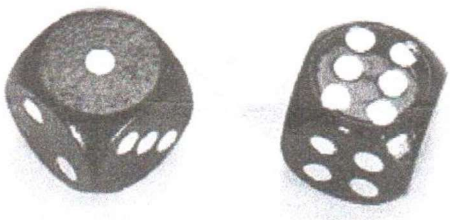
12. Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \tan x$, given $y(0) = 1$. *4*

13. Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3). *4*

Case-Study Based Question

14. A biased die is tossed and respective probabilities for various faces to turn up are the following :

Face	1	2	3	4	5	6
Probability	0·1	0·24	0·19	0·18	0·15	K



Based on the above information, answer the following questions :

(a) What is the value of K ? *2*

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4 ? *2*

2022 Compt.

SET-4

Series %BAB%/C

प्रश्न-पत्र कोड
Q.P. Code **65/B/6**

रोल नं.
Roll No. | | | | | | | | |

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **7** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Questions number **1** to **6** carry **2** marks each.*

1. Find the general solution of the differential equation : *2*

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

2. (a) Find : $\int \frac{x^3}{\sqrt{1 - x^8}} dx$ *2*

OR

(b) Find : $\int \frac{e^x . x}{(x + 1)^2} dx$ *2*

3. Two cards are drawn at random without replacement from a well-shuffled deck of 52 playing cards. Find the probability of getting both cards of the same colour. *2*

4. A bag contains 2 white, 2 red and 3 blue balls. Two balls are drawn at random from the bag one-by-one with replacement. Find the probability distribution of the number of white balls. *2*

5. Find the value of λ for which the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other, where $\vec{a} = 2\hat{i} - \hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$. *2*

6. Find the direction cosines of a line whose vector equation is given as $\vec{r} = (1-\lambda)\hat{i} + (2\lambda - 1)\hat{j} + (2\lambda + 3)\hat{k}$. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. Evaluate : $\displaystyle\int_{-4}^{4} |x+2|\, dx$ *3*

8. (a) Find the particular solution of the differential equation : *3*

$$(x \cos^2 (y/x) - y)\, dx + x\, dy = 0, \text{ given that } y = \frac{\pi}{4} \text{ when } x = 1.$$

OR

(b) Find the general solution of the differential equation : *3*

$$(x + y) \frac{dx}{dy} = 1$$

9. For vectors $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{c} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, determine (i) $\vec{b} \times \vec{c}$ and hence (ii) $\vec{a} \cdot (\vec{b} \times \vec{c})$. *3*

10. (a) Find the vector and the cartesian equation of the line passing through the points A(– 1, 3, 2) and B(– 4, 2, – 2). Also, find the value of λ, if the point P(5, 5, λ) lies on the line AB. *3*

OR

(b) Find the shortest distance between the lines : 3

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Find : $\int \frac{x}{(x+2)(3-2x)} dx$ 4

12. (a) Using integration, find the area of the region of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ bounded by the ordinates $x = 0$, $x = 2$. 4

OR

(b) Using integration, find the area of the region of the curve $y^2 = 4x$ bounded by the ordinates $x = 1$, $x = 4$. 4

13. Find the equation of the plane passing through the points P(4, 3, 4), Q(5, 3, 1) and R(7, 6, 2). Hence, find the distance of this plane from origin. 4

14. In a class, 5% of the boys and 10% of the girls have an IQ more than 150. In this class, 60% of the students are boys. One student is selected at random from the class.

Based on the above,

(a) Find the probability that the selected student has an IQ more than 150. 2

(b) If it is given that the selected student has IQ more than 150, find the probability that the student is a girl. 2

2022 Annual

Series : ABCD3/1

SET – 1

प्रश्न–पत्र कोड
Q.P. Code **65/1/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न–पत्र कोड को उत्तर–पुस्तिका के मुख–पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 8 हैं ।
- प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न–पत्र में 14 प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

*

गणित

MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *The question paper contains* ***three*** *Sections – Section* ***A, B*** *and* ***C****.*

(ii) ***Each*** *Section is* ***Compulsory****.*

(iii) ***Section – A*** *has* ***6*** *short answer type-****I*** *questions of* ***2*** *marks each.*

(iv) ***Section – B*** *has* ***4*** *short answer type-****II*** *questions of* ***3*** *marks each.*

(v) ***Section – C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question No.* ***14*** *is a case based problem with 2 subparts of* ***2*** *marks each.*

SECTION – A

Question numbers **1** to **6** carry **2** marks each.

1. Find the sum of the order and the degree of the differential equation :

$$\left(x+\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2+1$$ **2**

2. In a parallelogram PQRS, $\overrightarrow{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{PS} = -\hat{i} - 2\hat{k}$. Find $|\overrightarrow{PR}|$ and $|\overrightarrow{QS}|$. **2**

3. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right)=\frac{\pi}{4}$, then find $F(x)$. **2**

OR

(b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.

4. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of $P(B/A)$. **2**

5. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X. **2**

6. Find the values of λ, for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units. **2**

SECTION – B

Question numbers **7** to **10** carry **3** marks each.

7. (a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$. **3**

OR

(b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

8. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3). **3**

9. (a) Find : $\displaystyle\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$. **3**

OR

(b) Evaluate : $\displaystyle\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(4 + \sin x)}\, dx$.

10. Find the particular solution of the differential equation $x\dfrac{dy}{dx} + x\cos^2\left(\dfrac{y}{x}\right) = y$; given that when $x = 1$, $y = \dfrac{\pi}{4}$. **3**

SECTION – C

Question numbers **11** to **14** carry **4** marks each.

11. (a) Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \leq 36, 2x + 3y \geq 6\}$. **4**

OR

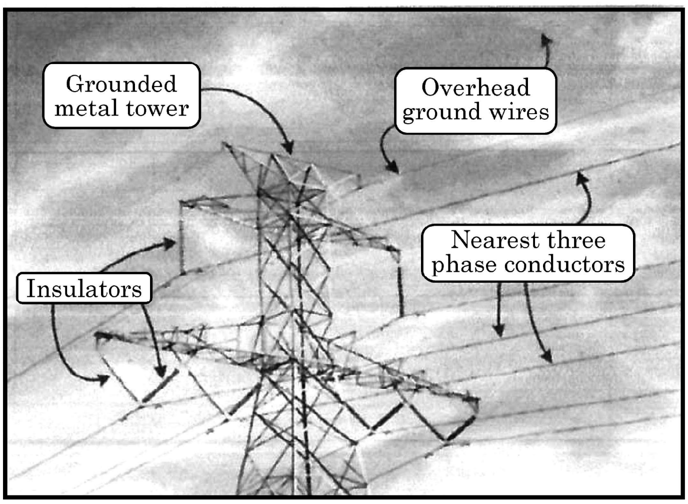
(b) Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x-axis.

12. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace. **4**

13. Evaluate : $\int_{0}^{\pi} \frac{x}{1+\sin x}\, dx$. **4**

CASE BASED / DATA BASED QUESTION

14. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer. **2**

(ii) Find the point of intersection of the lines l_1 and l_2. **2**

2022 Annual

Series : ABCD3/1 | **SET – 2**

प्रश्न-पत्र कोड
Q.P. Code **65/1/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
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गणित
MATHEMATICS

*निर्धारित समय : **2** घण्टे* | *अधिकतम अंक : **40***
*Time allowed : **2** hours* | *Maximum Marks : **40***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *The question paper contains* ***three*** *Sections – Section* ***A, B*** *and* ***C****.*

(ii) ***Each*** *Section is* ***Compulsory****.*

(iii) ***Section – A*** *has* ***6*** *short answer type-****I*** *questions of* ***2*** *marks each.*

(iv) ***Section – B*** *has* ***4*** *short answer type-****II*** *questions of* ***3*** *marks each.*

(v) ***Section – C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question No.* ***14*** *is a case based problem with 2 subparts of* ***2*** *marks each.*

SECTION – A

Question numbers **1** to **6** carry **2** marks each.

1. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find F($x$). **2**

OR

(b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.

2. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of P(B/A). **2**

3. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X. **2**

4. Find the values of λ, for which the distance of point (2, 1, λ) from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units. **2**

5. Solve the differential equation : **2**

$\log\left(\frac{dy}{dx}\right) = x - y.$

6. In a parallelogram PQRS, $\overrightarrow{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{PS} = -\hat{i} - 2\hat{k}$. Find $|\overrightarrow{PR}|$ and $|\overrightarrow{QS}|$. **2**

SECTION – B

Question numbers **7** to **10** carry **3** marks each.

7. Find the particular solution of the differential equation $x\frac{dy}{dx} + x\cos^2\left(\frac{y}{x}\right) = y$; given that when $x = 1$, $y = \frac{\pi}{4}$. **3**

8. (a) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$. **3**

OR

(b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

9. (a) Find : $\int \frac{x^3 + x}{x^4 - 9}\, dx$. **3**

OR

(b) Evaluate, using properties :

$$\int_{-\pi}^{\pi} (3\sin x - 2)^2\, dx.$$

10. Find the co-ordinates of the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ which is at a distance of 5 units from the point (1, 3, 3). **3**

SECTION – C

Question numbers **11** to **14** carry **4** marks each.

11. Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x}\, dx$. **4**

12. A man is known to speak truth 7 out of 10 times. He threw a pair of dice and reports that doublet appeared. Find the probability that it was actually a doublet. **4**

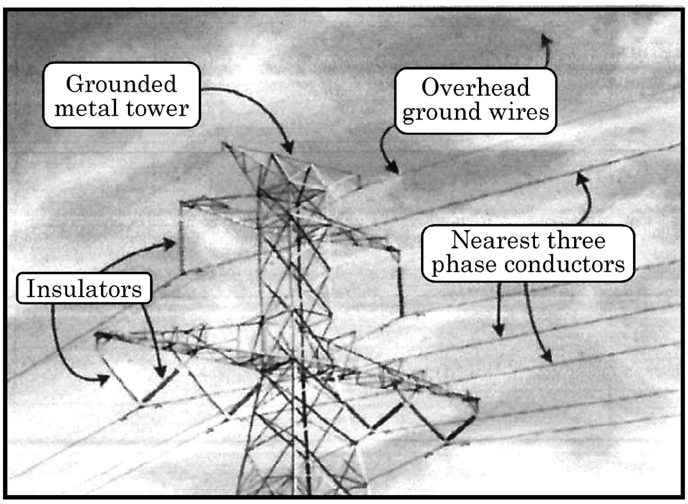
13. (a) Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \leq 36, 2x + 3y \geq 6\}$. **4**

OR

(b) Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x-axis.

CASE BASED / DATA BASED QUESTION

14. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer. **2**

(ii) Find the point of intersection of the lines l_1 and l_2. **2**

2022 Annual

Series : ABCD3/1 **SET – 3**

प्रश्न–पत्र कोड
Q. P. Code **65/1/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न–पत्र कोड को उत्तर–पुस्तिका के मुख–पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
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- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

*

गणित

MATHEMATICS

*निर्धारित समय : **2** घण्टे* *अधिकतम अंक : **40***

*Time allowed : **2** hours* *Maximum Marks : **40***

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *The question paper contains **three** Sections – Section **A**, **B** and **C**.*

(ii) ***Each** Section is **Compulsory**.*

(iii) ***Section – A** has **6** short answer type-**I** questions of **2** marks each.*

(iv) ***Section – B** has **4** short answer type-**II** questions of **3** marks each.*

(v) ***Section – C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question No. **14** is a case based problem with 2 subparts of **2** marks each.*

SECTION – A

Question numbers **1** to **6** carry **2** marks each.

1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of P(B/A). **2**

2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X. **2**

3. Find the sum of the order and the degree of the differential equation :

$$\left(x+\frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 + 1$$ **2**

4. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$. **2**

OR

(b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.

5. Find the values of λ, for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units. **2**

6. If $\vec{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\vec{a} + \vec{b})$ is perpendicular to the vector $(\vec{a} - \vec{b})$, then find all the possible values of y. **2**

SECTION – B

Question numbers **7** to **10** carry **3** marks each.

7. (a) Find : $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$. **3**

OR

(b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)\,(4 + \sin x)}\, dx$.

8. Find the co-ordinates of point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through points $\left(\frac{7}{2}, 0, 0\right)$, (0, 7, 0) and (0, 0, 7). **3**

9. Solve the differential equation : $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$. **3**

10. (a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$. **3**

OR

(b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

SECTION – C

Question numbers **11** to **14** carry **4** marks each.

11. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace. **4**

12. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$. **4**

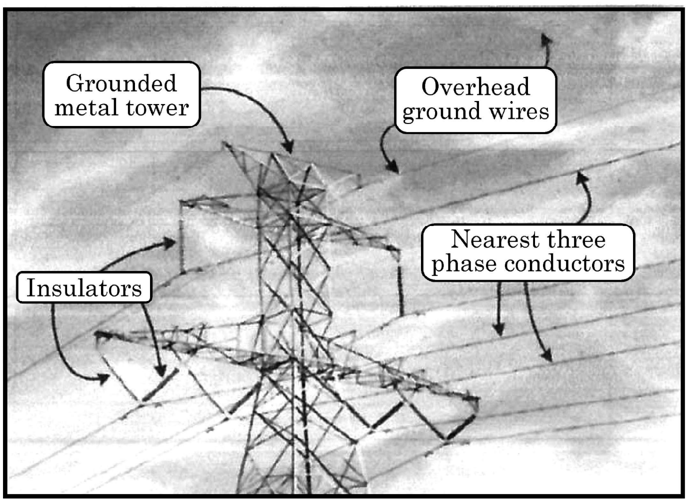
13. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$. **4**

OR

Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$, semi-circle $y = \sqrt{4-x^2}$ and x-axis in first quadrant.

CASE BASED / DATA BASED QUESTION

14. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer. **2**

(ii) Find the point of intersection of the lines l_1 and l_2. **2**

2022 Annual

Series ABCD1/2

Set No. 1

प्रश्न-पत्र कोड
Q.P. Code **65/2/1**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$. *2*

2. (a) Find : *2*

$$\int \frac{\sin 3x}{\sin x}\, dx$$

OR

(b) Evaluate : *2*

$$\int_0^{\frac{1}{2}\log 3} \frac{e^x}{e^{2x}+1}\, dx$$

3. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. *2*

4. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3. *2*

5. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved. *2*

6. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is – 2. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. ABCD is a parallelogram such that $\vec{AC} = \hat{i} + \hat{j}$ and $\vec{BD} = 2\hat{i} + \hat{j} + \hat{k}$. Find $\vec{AB}$ and $\vec{AD}$. Also, find the area of the parallelogram ABCD. *3*

8. (a) Evaluate : *3*

$$\int_0^1 \tan^{-1} x \, dx$$

OR

(b) Find : *3*

$$\int \frac{2x}{x^2 + 3x + 2} \, dx$$

9. Find the particular solution of the differential equation $(y + 3x^2) \frac{dx}{dy} = x$, given that y = 1, when x = 1. *3*

10. (a) Find the equation of the plane passing through points (2, 1, 0), (3, – 2, – 2) and (1, 1, – 7). Also, obtain its distance from the origin. *3*

OR

(b) Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. *4*

12. Evaluate : *4*

$$\int_0^1 x(1-x)^n \, dx$$

13. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. *4*

OR

(b) If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$. *4*

Case-Study Based Question

14. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails. *2*

(b) Find the probability of getting at least one head in three tosses of such a coin. *2*

2022 Annual

Series ABCD1/2

Set No. 2

प्रश्न-पत्र कोड
Q.P. Code 65/2/2

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
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गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Questions number **1** to **6** carry **2** marks each.*

1. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3. *2*

2. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. *2*

3. (a) Find : *2*

$$\int \frac{\sin 3x}{\sin x}\, dx$$

OR

(b) Evaluate : *2*

$$\int_0^{\frac{1}{2}\log 3} \frac{e^x}{e^{2x}+1}\, dx$$

4. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved. *2*

5. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is – 2. *2*

6. Find the particular solution of the differential equation $\frac{dy}{dx} = 2y^2$, given y = 1 when x = 1. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. (a) Find the equation of the plane passing through points (2, 1, 0), (3, – 2, – 2) and (1, 1, – 7). Also, obtain its distance from the origin. *3*

OR

(b) Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$. *3*

8. Find the general solution of the differential equation

$$x\frac{dy}{dx} - y + x\sin\frac{y}{x} = 0.$$ *3*

9. (a) Evaluate : *3*

$$\int_0^1 \tan^{-1} x \, dx$$

OR

(b) Find : *3*

$$\int \frac{2x}{x^2 + 3x + 2} dx$$

10. Using vectors, find the value of 'b' if the points A(–1, –1, 2), B(2, b, 5) and C(3, 11, 6) are collinear . Also, determine the ratio in which the point B divides the line-segment AC internally. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. *4*

OR

(b) If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$. *4*

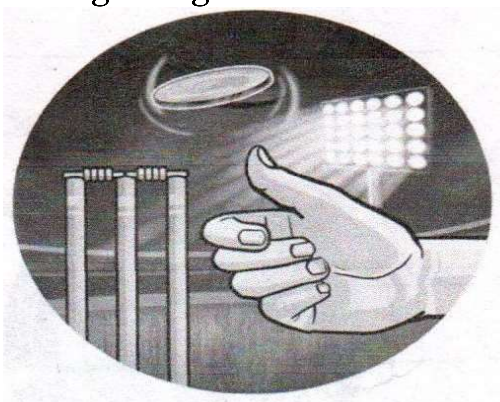
12. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. *4*

13. Evaluate : *4*

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$$

Case-Study Based Question

14. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.



Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails. *2*

(b) Find the probability of getting at least one head in three tosses of such a coin. *2*

2022 Annual

Series ABCD1/2 **Set No. 3**

प्रश्न-पत्र कोड
Q.P. Code **65/2/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
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गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is – 2. *2*

2. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved. *2*

3. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3. *2*

4. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. *2*

5. Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$. *2*

6. (a) Find : *2*

$$\int \frac{\sqrt{\cot x}}{\sin x \cos x} \, dx$$

OR

(b) Find : *2*

$$\int \frac{1}{x\,(x^2 + 4)} \, dx$$

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. (a) Evaluate : *3*

$$\int_0^1 \tan^{-1} x \, dx$$

OR

(b) Find : *3*

$$\int \frac{2x}{x^2 + 3x + 2} \, dx$$

8. (a) Find the equation of the plane passing through points (2, 1, 0), (3, – 2, – 2) and (1, 1, – 7). Also, obtain its distance from the origin. *3*

OR

(b) Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$. *3*

9. Solve the following differential equation : *3*

$$(1 + e^{y/x})\, dy + e^{y/x}\left(1 - \frac{y}{x}\right) dx = 0$$

10. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, then find the vector $\vec{c}$, given that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 4$. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Evaluate : *4*

$$\int_0^1 x\,(1 - x)^n \, dx$$

12. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. *4*

OR

(b) If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$. *4*

13. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the zx-plane. *4*

Case-Study Based Question

14. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails. *2*

(b) Find the probability of getting at least one head in three tosses of such a coin. *2*

2022 Annual

Series : ABCD4/3

SET – 1

प्रश्न-पत्र कोड
Q.P. Code **65/3/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

*

गणित
MATHEMATICS

*निर्धारित समय : **2** घण्टे* — *अधिकतम अंक : **40***

*Time allowed : **2** hours* — *Maximum Marks : **40***

General Instructions :

(i) *This question paper contains* ***three*** *Sections – Section* ***A, B*** *and* ***C****.*

(ii) *Each Section is compulsory.*

(iii) *Section–****A*** *has* ***6*** *short answer type-****I*** *questions of* ***2*** *marks each.*

(iv) *Section–****B*** *has* ***4*** *short answer type-****II*** *questions of* ***3*** *marks each.*

(v) *Section–****C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Q.* ***14*** *is a case study based problem with* ***2*** *sub-parts of* ***2*** *marks each.*

SECTION – A

Question Nos. **1** to **6** carry **2** marks each.

1. Find : $\int \frac{dx}{x^2 - 6x + 13}$ **2**

2. Find the general solution of the differential equation : $e^{dy/dx} = x^2$. **2**

3. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **2**

4. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ. **2**

5. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards. **2**

6. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. **2**

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

SECTION – B

Question Nos. **7** to **10** carry **3** marks each.

7. Evaluate : $\int_{0}^{2\pi} \frac{dx}{1+e^{\sin x}}$ **3**

8. Find the particular solution of the differential equation $x \frac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$. **3**

OR

Find the general solution of the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.

9. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram. **3**

OR

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that the vector $(\vec{a} + \lambda \vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ.

10. Show that the lines : **3**
$\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z - 1$ are coplanar.

SECTION – C

Question Nos. **11** to **14** carry **4** marks each.

11. Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration. **4**

12. Find : $\int \frac{x^2}{(x^2+1)(3x^2+4)} dx$ **4**

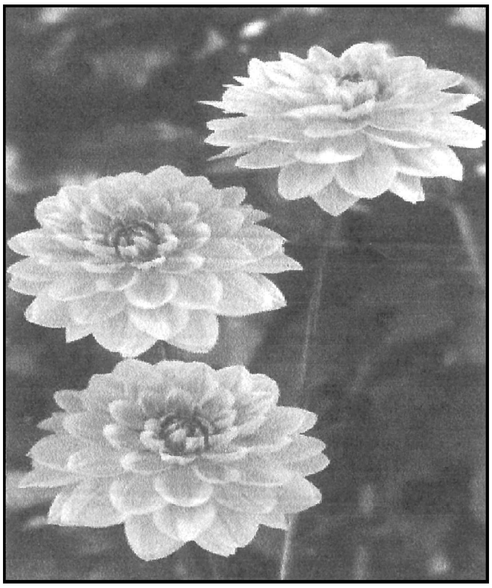
OR

Evaluate : $\int_{-2}^{1} \sqrt{5-4x-x^2}\, dx$

13. Find the distance of the point (1, –2, 9) from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$. **4**

Case Study Problem :

14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.



Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate; **2**

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. **2**

2022 Annual

Series : ABCD4/3

SET – 2

प्रश्न–पत्र कोड Q.P. Code 65/3/2

रोल नं.
Roll No.

परीक्षार्थी प्रश्न–पत्र कोड को उत्तर–पुस्तिका के मुख–पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

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गणित
MATHEMATICS

*निर्धारित समय : **2** घण्टे* | *अधिकतम अंक : **40***

*Time allowed : **2** hours* | *Maximum Marks : **40***

General Instructions :

(i) *This question paper contains* **three** *Sections – Section* ***A, B*** *and* ***C****.*

(ii) *Each Section is compulsory.*

(iii) *Section–****A*** *has* ***6*** *short answer type-****I*** *questions of* ***2*** *marks each.*

(iv) *Section–****B*** *has* ***4*** *short answer type-****II*** *questions of* ***3*** *marks each.*

(v) *Section–****C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Q.* ***14*** *is a case study based problem with* ***2*** *sub-parts of* ***2*** *marks each.*

SECTION – A

Question Nos. **1** to **6** carry **2** marks each.

1. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **2**

2. Find the general solution of the differential equation : $\log\left(\frac{dy}{dx}\right) = ax + by$. **2**

3. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards. **2**

4. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. **2**

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

5. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ. **2**

6. Find : $\int \frac{dx}{x^2 - 6x + 13}$ **2**

SECTION – B

Question Nos. **7** to **10** carry **3** marks each.

7. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$, then show that $\vec{b} = \vec{c}$. **3**

OR

If $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.

8. Evaluate : $\displaystyle\int_{-1}^{2} |x^3 - x| \, dx$ **3**

9. Show that the lines : **3**

$\dfrac{1-x}{2} = \dfrac{y-3}{4} = \dfrac{z}{-1}$ and $\dfrac{x-4}{3} = \dfrac{2y-2}{-4} = z - 1$ are coplanar.

10. Find the particular solution of the differential equation $x\dfrac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$. **3**

OR

Find the general solution of the differential equation $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$.

SECTION – C

Question Nos. **11** to **14** carry **4** marks each.

11. Find : $\displaystyle\int \frac{x^2}{(x^2+1)(3x^2+4)}\, dx$ **4**

OR

Evaluate : $\displaystyle\int_{-2}^{1} \sqrt{5 - 4x - x^2}\, dx$

12. Using integration, find the area of the region bounded by the curves $x^2 + y^2 = 4$, $x = \sqrt{3}\, y$ and x-axis lying in the first quadrant. **4**

13. Find the distance of the point (1, –2, 9) from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$. **4**

Case Study Problem :

14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate; **2**

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. **2**

2022 Annual

Series : ABCD4/3

SET – 3

प्रश्न–पत्र कोड
Q.P. Code 65/3/3

रोल नं.
Roll No.

परीक्षार्थी प्रश्न–पत्र कोड को उत्तर–पुस्तिका के मुख–पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. sCode on the title page of the answer-

- कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **8** हैं ।
- प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

- Please check that this question paper contains **8** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
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गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*
Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

(i) *This question paper contains* ***three*** *Sections – Section* ***A, B*** *and* ***C****.*

(ii) *Each Section is compulsory.*

(iii) *Section–**A** has **6** short answer type-**I** questions of **2** marks each.*

(iv) *Section–**B** has **4** short answer type-**II** questions of **3** marks each.*

(v) *Section–**C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Q. **14** is a case study based problem with **2** sub-parts of **2** marks each.*

SECTION – A

Question Nos. **1** to **6** carry **2** marks each.

1. If the distance of the point (1, 1, 1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ. **2**

2. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. **2**

3. Find the general solution of the differential equation : $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^{x} + e^{-x}}$ **2**

4. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards. **2**

5. Find : $\int \frac{dx}{x^2 - 6x + 13}$ **2**

6. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. **2**

OR

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

SECTION – B

Question Nos. **7** to **10** carry **3** marks each.

7. Find the shortest distance between the following lines : **3**

$\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$.

8. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram. **3**

OR

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that the vector $(\vec{a} + \lambda\vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ.

9. Find the particular solution of the differential equation $x\dfrac{dy}{dx} - y = x^2 \cdot e^x$, given $y(1) = 0$. **3**

OR

Find the general solution of the differential equation $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$.

10. Evaluate : $\displaystyle\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\,dx$ **3**

SECTION – C

Question Nos. **11** to **14** carry **4** marks each.

11. Find the distance of the point (1, –2, 9) from the point of intersection of the line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$. **4**

12. Find : $\displaystyle\int \frac{x^2}{(x^2+1)(3x^2+4)}\,dx$ **4**

OR

Evaluate : $\displaystyle\int_{-2}^{1} \sqrt{5 - 4x - x^2}\,dx$

13. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration. **4**

Case Study Problem :

14. A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate; **2**

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates. **2**

2022 Annual

Series ABDC2/4 **Set No. 1**

प्रश्न-पत्र कोड
Q.P. Code 65/4/1

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ* ***7*** *हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में* ***14*** *प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains* ***7*** *printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains* ***14*** *questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे *अधिकतम अंक : 40*

Time allowed : 2 hours *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Questions number **1** to **6** carry **2** marks each.*

1. A bag contains 3 red and 4 white balls. Three balls are drawn at random, one-by-one without replacement from the bag. If the first ball drawn is red in colour, then find the probability that the remaining two balls drawn are also red in colour. *2*

2. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer. *2*

3. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane. *2*

4. (a) If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{b}| = 5$, then find the value of $|\vec{a}|$. *2*

OR

(b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes. *2*

5. Find the general solution of the differential equation

$$\sec^2 x \,.\, \tan y \, dx + \sec^2 y \,.\, \tan x \, dy = 0.$$ *2*

6. Evaluate : *2*

$$\int_0^1 x^2 e^x \, dx$$

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. Find the area of the region $\{(x, y) : x^2 \leq y \leq x + 2\}$, using integration. *3*

8. (a) Find : *3*

$$\int \frac{1}{e^x + 1} \, dx$$

OR

(b) Evaluate : *3*

$$\int_1^4 \{ |x| + |3 - x| \} \, dx$$

9. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude, then prove that the vector $(2\vec{a} + \vec{b} + 2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$. Also, find the angle between $\vec{a}$ and $(2\vec{a} + \vec{b} + 2\vec{c})$. *3*

10. (a) If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. *3*

OR

(b) Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Find the equations of the planes passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) = 6$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which are at a distance of 1 unit from the origin. *4*

12. (a) Find the particular solution of the differential equation $x\dfrac{dy}{dx} + y + \dfrac{1}{1+x^2} = 0$, given that $y(1) = 0$. *4*

OR

(b) Find the general solution of the differential equation $x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$. *4*

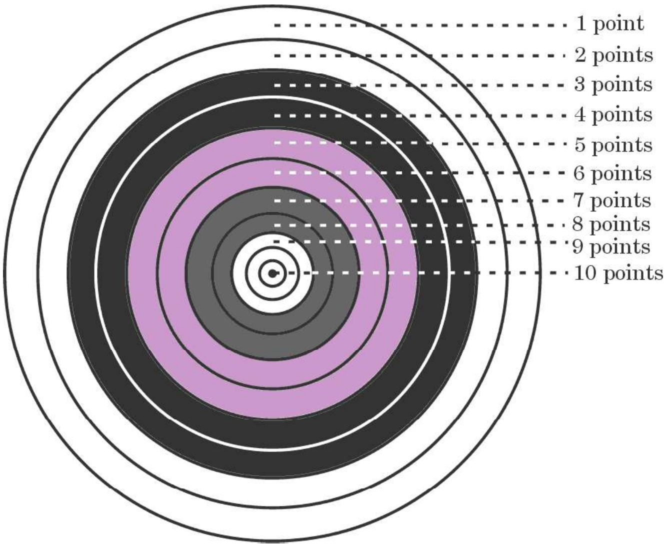
13. Evaluate : *4*

$$\int_0^{\pi/2} (2\log\cos x - \log\sin 2x)\,dx$$

Case-Study Based Question

14. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.



Based on the above information, answer the following questions :

If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points. *2*

(b) both of them earn 10 points. *2*

2022 Annual

Series ABDC2/4

Set No. 2

प्रश्न-पत्र कोड
Q.P. Code 65/4/2

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane. *2*

2. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer. *2*

3. (a) If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{b}| = 5$, then find the value of $|\vec{a}|$. *2*

OR

(b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes. *2*

4. Find the general solution of the differential equation

$$\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0.$$ *2*

5. Evaluate : *2*

$$\int_0^1 x^2 e^x \, dx$$

6. There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. Using integration, find the area of the region $\{(x, y) : y^2 \leq x \leq y\}$. *3*

8. (a) If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. *3*

OR

(b) Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not. *3*

9. (a) Find : *3*

$$\int \frac{1}{e^x + 1} \, dx$$

OR

(b) Evaluate : *3*

$$\int_1^4 \{ |x| + |3 - x| \} \, dx$$

10. If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = \cot\left(\frac{\alpha}{2}\right)$. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. (a) Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$, given that $y(1) = 0$. *4*

OR

(b) Find the general solution of the differential equation $x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$. *4*

12. Evaluate : *4*

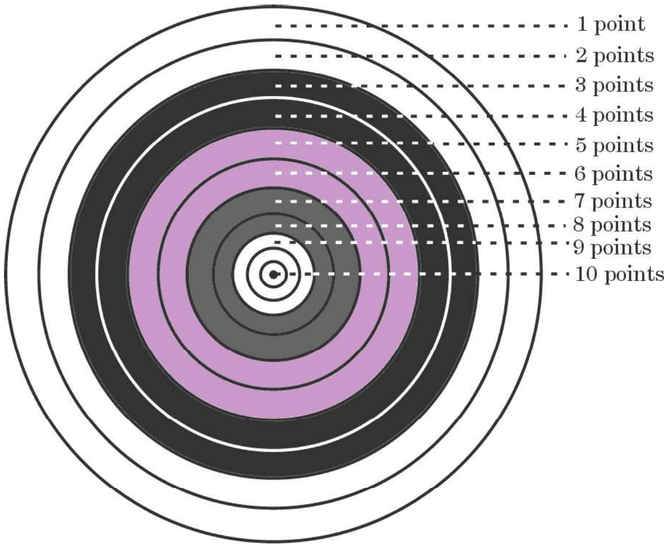
$$\int_0^{\pi} \frac{x}{9\sin^2 x + 16\cos^2 x}\,dx$$

13. Find the equations of the planes passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i} + 3\hat{j}) = 6$ and $\vec{r}\cdot(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which are at a distance of 1 unit from the origin. *4*

Case-Study Based Question

14. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.



Based on the above information, answer the following questions :

If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points. *2*

(b) both of them earn 10 points. *2*

2022 Annual

Series ABDC2/4

Set No. 3

प्रश्न-पत्र कोड
Q.P. Code **65/4/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
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- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **three** sections – **Section A, B** and **C**.*

(ii) *Each section is **compulsory**.*

(iii) ***Section A** has **6** short answer type I questions of **2** marks each.*

(iv) ***Section B** has **4** short answer type II questions of **3** marks each.*

(v) ***Section C** has **4** long answer type questions of **4** marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no. **14** is a case-study based question with 2 sub-parts of **2** marks each.*

SECTION A

*Questions number **1** to **6** carry **2** marks each.*

1. (a) If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 400$ and $|\vec{b}| = 5$, then find the value of $|\vec{a}|$. *2*

OR

(b) Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes. *2*

2. Evaluate : *2*

$$\int_0^1 x^2 e^x \, dx$$

3. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner. *2*

4. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
P(X)	K	6K	9K

(a) Find the value of K.

(b) Is the coin tossed biased or unbiased ? Justify your answer. *2*

5. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane. *2*

6. Find the general solution of the differential equation

$$\sec^2 x \,.\, \tan y \, dx + \sec^2 y \,.\, \tan x \, dy = 0.$$ *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration. *3*

8. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ and the projection of vector $\vec{c} + \lambda \vec{b}$ on vector $\vec{a}$ is $2\sqrt{6}$, then find the value of λ. *3*

9. (a) If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. *3*

OR

(b) Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not. *3*

10. (a) Find : *3*

$$\int \frac{1}{e^x + 1} \, dx$$

OR

(b) Evaluate : *3*

$$\int_1^4 \{ |x| + |3 - x| \} \, dx$$

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Evaluate : *4*

$$\int_{-3}^{3} \frac{x^4}{1 + e^x}\, dx$$

12. Find the equations of the planes passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) = 6$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which are at a distance of 1 unit from the origin. *4*

13. (a) Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$, given that $y(1) = 0$. *4*

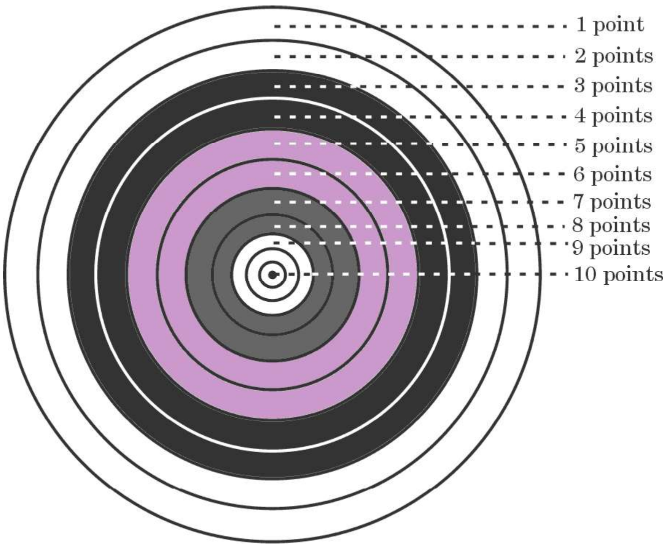
OR

(b) Find the general solution of the differential equation $x(y^3 + x^3)\,dy = (2y^4 + 5x^3y)\,dx$. *4*

Case-Study Based Question

14. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.



Based on the above information, answer the following questions :
If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points. *2*

(b) both of them earn 10 points. *2*

2022 Annual

Series ABCD5/5

SET No. 1

प्रश्न पत्र कोड
Q.P. Code 65/5/1

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 7 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains **7** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे
Time allowed : 2 hours

अधिकतम अंक : 40
Maximum Marks : 40

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. *This question paper contains* ***three*** *Sections-* ***A, B*** *and* ***C.***
2. ***Each*** *section is compulsory.*
3. ***Section-A*** *has 6 short-answer type-I questions of* ***2*** *marks each.*
4. ***Section-B*** *has 4 short-answer type-II questions of* ***3*** *marks each.*
5. ***Section-C*** *has 4 long-answer type questions of* ***4*** *marks each.*
6. *There is an internal choice in some questions.*
7. *Question 14 is a case study based question with* ***two*** *subparts of* ***2*** *marks each.*

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. Find : $\int \frac{dx}{\sqrt{4x-x^2}}$ 2

2. Find the general solution of the following differential equation : 2

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

3. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. 2

Find the probability distribution of X.

4. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{a}.\vec{b}=1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $\left|\vec{b}\right|$ 2

5. If a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$. 2

6. (a) Events A and B are such that 2

$P(A)=\frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\overline{A}\cup\overline{B}) = \frac{1}{4}$

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour. 2

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. Evaluate : 3

$$\int_0^{\pi/4} \frac{dx}{1+\tan x}$$

8. (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right| = \left|\vec{b}\right|$, then prove that $\left(\vec{a}+2\vec{b}\right)$ is perpendicular to $\vec{a}$. 3

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that 3

$$\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a}-\vec{b}\right|.$$

9. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right) = 10$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4 = 0$ and passing through the point (–2, 3, 1). 3

10. (a) Find : 3

$$\int e^x . \sin 2x \, dx$$

OR

(b) Find : 3

$$\int \frac{2x}{\left(x^2+1\right)\left(x^2+2\right)} dx$$

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. Three persons A, B and C apply for a job of manager in a private company. Chances of their selection are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce changes to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A. 4

12. Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration. 4

13. (a) Solve the following differential equation : 4

$$\left(y - \sin^2 x\right)dx + \tan x\,dy = 0$$

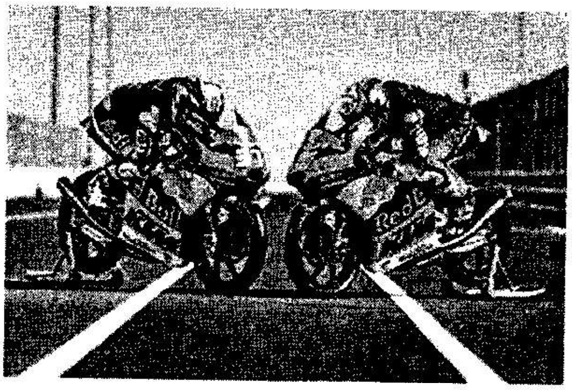
OR

(b) Find the general solution of the differential equation : 4

$$\left(x^3 + y^3\right)dy = x^2 y\,dx$$

Case Study Based Question

14. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ respectively. 2×2=4



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines. 2

(b) Find the point at which the motorcycles may collide. 2

Series ABCD5/5

2022 Annual

SET No. 2

प्रश्न पत्र कोड
Q.P. Code **65/5/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 7 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains **7** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे
Time allowed : 2 hours

अधिकतम अंक : 40
Maximum Marks : 40

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. *This question paper contains **three** Sections- **A, B** and **C**.*
2. ***Each** section is compulsory.*
3. ***Section-A** has 6 short-answer type-I questions of **2** marks each.*
4. ***Section-B** has 4 short-answer type-II questions of **3** marks each.*
5. ***Section-C** has 4 long-answer type questions of **4** marks each.*
6. *There is an internal choice in some questions.*
7. *Question 14 is a case study based question with **two** subparts of **2** marks each.*

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. Find the vector equation of a line passing through a point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. 2

2. Find : $\int \frac{dx}{\sqrt{4x - x^2}}$ 2

3. Find the general solution of the following differential equation : 2

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

4. (a) Events A and B are such that 2

$P(A)=\frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour. 2

5. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that 2
$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.
Find the probability distribution of X.

6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $\left|\vec{b}\right|$ 2

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. (a) Find : 3

$$\int e^x . \sin 2x \, dx$$

OR

(b) Find : 3

$$\int \frac{2x}{(x^2+1)(x^2+2)} dx$$

8. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 10$ and $\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ and passing through the point (–2, 3, 1). 3

9. (a) Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{a}.\hat{n} = 0$ and $\vec{b}.\hat{n} = 0$, then find $\left|\vec{c}.\hat{n}\right|$. 3

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle 30° to each other, then find the area of the parallelogram with $\left(\vec{a} + 3\vec{b}\right)$ and $\left(3\vec{a} + \vec{b}\right)$ as adjacent sides. 3

10. Evaluate : 3

$$\int_0^{\pi/2} \frac{1}{1+(\tan x)^{2/3}} dx$$

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. (a) Solve the following differential equation : 4

$$\left(y-\sin^2 x\right)dx + \tan x\, dy = 0$$

OR

(b) Find the general solution of the differential equation : 4

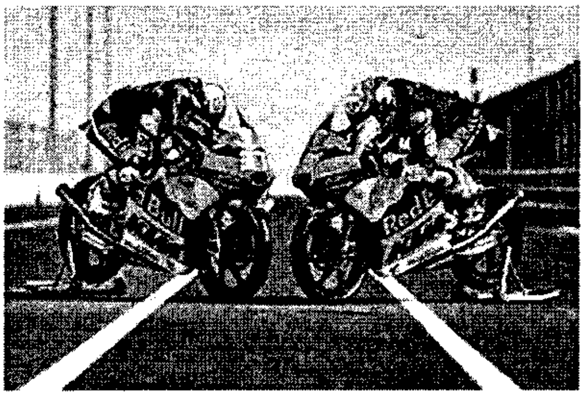
$$\left(x^3+y^3\right)dy = x^2 y\, dx$$

12. Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration. 4

13. In a factory, machine A produces 30% of total output, machine B produces 25% and the machine C produces the remaining output. The defective items produced by machines A, B and C are 1%, 1.2%, 2% respectively. An item is picked at random from a day's output and found to be defective. Find the probability that it was produced by machine B? 4

Case Study Based Question

14. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r} = \left(3\hat{i}+3\hat{j}\right)+\mu\left(2\hat{i}+\hat{j}+\hat{k}\right)$ respectively. 2×2=4



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines. 2

(b) Find the point at which the motorcycles may collide. 2

Series ABCD5/5

2022 Annual

SET No. 3

प्रश्न पत्र कोड
Q.P. Code **65/5/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 7 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 7 printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे — *अधिकतम अंक : 40*

Time allowed : 2 hours — *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. *This question paper contains **three** Sections- **A, B** and **C**.*
2. ***Each** section is compulsory.*
3. ***Section-A** has 6 short-answer type-I questions of **2** marks each.*
4. ***Section-B** has 4 short-answer type-II questions of **3** marks each.*
5. ***Section-C** has 4 long-answer type questions of **4** marks each.*
6. *There is an internal choice in some questions.*
7. *Question 14 is a case study based question with **two** subparts of **2** marks each.*

SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. The Cartesian equation of a line AB is : 2

$$\frac{2x-1}{12}=\frac{y+2}{2}=\frac{z-3}{3}$$

Find the direction cosines of a line parallel to line AB.

2. Find : $\int \frac{dx}{\sqrt{4x-x^2}}$ 2

3. (a) Events A and B are such that 2

$P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$ and $P(\bar{A}\cup\bar{B})=\frac{1}{4}$

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour. 2

4. Find the general solution of the following differential equation : 2

$$\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$$

5. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{a}.\vec{b}=1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $\left|\vec{b}\right|$ 2

6. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that 2
$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.
Find the probability distribution of X.

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right| = \left|\vec{b}\right|$, then prove that 3
$\left(\vec{a}+2\vec{b}\right)$ is perpendicular to $\vec{a}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that 3

$$\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a}-\vec{b}\right|.$$

8. (a) Find : 3

$$\int e^x . \sin 2x \, dx$$

OR

(b) Find : 3

$$\int \frac{2x}{\left(x^2+1\right)\left(x^2+2\right)} dx$$

9. Evaluate : $\displaystyle\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}} dx$ 3

10. Find the distance of the point (2, 3, 4) measured along the line $\frac{x-4}{3}=\frac{y+5}{6}=\frac{z+1}{2}$ from the plane $3x+2y+2z+5 = 0$. 3

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. Find the area bounded by the curves $y = |x-1|$ and $y = 1$, using integration. 4

12. (a) Solve the following differential equation : 4

$$\left(y - \sin^2 x\right)dx + \tan x\,dy = 0$$

OR

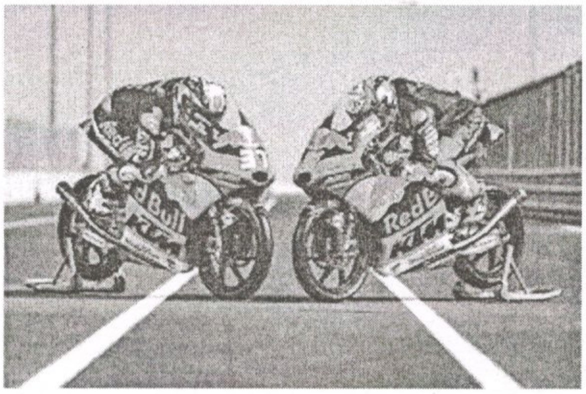
(b) Find the general solution of the differential equation : 4

$$\left(x^3 + y^3\right)dy = x^2 y\,dx$$

13. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II. 4

Case Study Based Question

14. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ respectively. 2×2=4



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines. 2

(b) Find the point at which the motorcycles may collide. 2

SET-5

Series %BAB%

2022 Annual

प्रश्न-पत्र कोड
Q.P. Code **65/B/5**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **7** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. (a) Find : $\int \frac{1}{\sqrt{12 + 4x - x^2}} \, dx$ *2*

OR

(b) Find : $\int \frac{xe^x}{(x + 4)^5} \, dx$ *2*

2. Find the general solution of the following differential equation : *2*

$$(4 + y^2)(3 + \log x)\, dx + x\, dy = 0$$

3. If $|\vec{a}| = 3$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{a} \cdot \vec{b} = 6$, then find the value of $|\vec{a} \times \vec{b}|$. *2*

4. Find the direction cosines of a line whose cartesian equation is given as $3x + 1 = 6y - 2 = 1 - z$. *2*

5. In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up. *2*

6. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement. *2*

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. Evaluate : $\int_{0}^{\pi/3} |\cos 3x| \, dx$ *3*

8. (a) Find the general solution of the following differential equation : *3*

$$2x\, e^{y/x}\, dy + (x - 2y\, e^{y/x})\, dx = 0$$

OR

(b) Find the particular solution of the differential equation $(2x^2 + y) \cdot \frac{dx}{dy} = x$; given that $y = 2$ when $x = 1$. *3*

9. Using vectors, find the area of the triangle with vertices $A(-1, 0, -2)$, $B(0, 2, 1)$ and $C(-1, 4, 1)$. *3*

10. (a) Find the shortest distance between the lines

$$\vec{r} = (\lambda + 1)\,\hat{i} + (\lambda + 4)\,\hat{j} - (\lambda - 3)\,\hat{k}, \text{ and}$$

$$\vec{r} = (3 - \mu)\,\hat{i} + (2\mu + 2)\,\hat{j} + (\mu + 6)\,\hat{k}.$$ *3*

OR

(b) Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5)$, $B(-5, -2, 3)$ and $C(2, 4, -5)$. *3*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Find : $\int \frac{x^2 + x + 1}{(x+1)(x^2+4)} dx$ *4*

12. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration. *4*

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration. *4*

13. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$. *4*

Case-Study Based Question

14. A laboratory blood test is 98% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0·4% of the healthy person tested. From a large population, it is given that 0·2% of the population actually has the disease.

Based on the above, answer the following questions :

(a) One person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result. *2*

(b) What is the probability that the person actually has the disease, given that his test result is positive ? *2*